Math 11, Multivariable Calculus
Written Homework 8

1. (Chapter 16.4, #22) Let D be a region bounded by a simple closed path C in the xy-plane. Use Green's Theorem to prove that the coordinates $(\bar{x}, \bar{y})$ of the centroid (the centroid is the center of mass of D, if we assume that D is a lamina of uniform density ρ and area A) of D are
$$\bar{x} = \frac{1}{2A} \int_C x^2 \, dy, \quad \bar{y} = -\frac{1}{2A} \int_C y^2 \, dx.$$

2. (Chapter 16.4, #12 modified) Let C be the arc of the curve $y = \cos x$ from $(-\pi/2, 0)$ to $(\pi/2, 0)$, and $\mathbf{F} = \langle x + y^2, e^{-y^2} + x^2 \rangle$ a vector field. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$. It is a wise person who uses Green's theorem.

3. (Chapter 16.5, #20) Is there a smooth vector field $\mathbf{G}$ on $\mathbb{R}^3$ such that $\nabla \times \mathbf{G} = \langle xyz, -y^2 z, yz^2 \rangle$? Explain.

4. (Chapter 16.5, #25.) Prove $\operatorname{div}(f\mathbf{F}) = f \operatorname{div} \mathbf{F} + \mathbf{F} \cdot \nabla f$ assuming that the appropriate partial derivatives exist and are continuous. Here $\mathbf{F} = \langle P, Q, R \rangle$ and P, Q, R, f are all scalar-valued functions of the variables x, y, z.

5. (Chapter 16.6, #36) Let $\mathbf{r}(u, v) = \langle \sin u, \cos u \sin v, \sin v \rangle$. Find an equation for the tangent plane to this surface at $u = \pi/6,\ v = \pi/6$.

6. (Chapter 16.6, #42) Find the surface area of the part of the cone $z = \sqrt{x^2 + y^2}$ that lies between the plane $y = x$ and the parabolic cylinder $y = x^2$.