Math 11, Multivariable Calculus
Written Homework 6

1. (Ch 15.1: 17,18)

   (a) Show that if f is a constant function of two variables (so $f(x, y) = k$ for some constant k) and $R = [a, b] \times [c, d]$, then

   $$\iint_R k \, dA = k(b - a)(d - c).$$

   (b) Use part (a) to show that

   $$0 \leq \iint_R \sin(\pi x) \cos(\pi y) \, dA \leq \frac{1}{32},$$

   where $R = [0, 1/4] \times [1/4, 1/2]$.

2. (Ch 15.2: 36) Find the average value of $f(x, y) = e^y \sqrt{x + e^y}$ over the rectangle $R = [0, 4] \times [0, 1]$.

3. (Ch 15.3: 54) Evaluate the following integral by interchanging the order of integration:

   $$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} \, dx \, dy.$$

4. (Ch 15.3) In evaluating a double integral over a region D, a sum of iterated integrals was obtained as follows:

   $$\iint_D f(x, y) \, dA = \int_0^2 \int_0^{\sqrt{y}} f(x, y) \, dx \, dy + \int_2^4 \int_{y-2}^{\sqrt{y}} f(x, y) \, dx \, dy.$$

   Sketch the region D and express the double integral as an iterated integral with reversed order of integration.

5. (Ch 15.7: 16) Evaluate the triple integral $\iiint_T xyz \, dV$, where T is the solid tetrahedron with vertices $(0, 0, 0), (1, 0, 0), (1, 1, 0), (1, 0, 1)$.

6. (Ch 15.7: 28) Sketch the solid whose volume is given by the following iterated integral, and compute the value of that volume:

   $$\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx \, dz \, dy.$$