Math 11, Multivariable Calculus
Written Homework 4

1. (14.6 #64a) The plane $y + z = 3$ intersects the cylinder $x^2 + y^2 = 5$ in an ellipse. Find parametric equations for the tangent line to this ellipse at the point $(1, 2, 1)$.

2. (14.7 #47) Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 6$.

3. (14.7 #52) A rectangular building is being designed to minimize heat loss. The east and west walls lose heat at a rate of 10 units/m^2 per day, the north and south walls at a rate of 8 units/m^2 per day, the floor at 1 unit/m^2 per day, and the roof at a rate of 5 units/m^2 per day. Each wall must be at least 30m long, the height must be at least 4m, and the volume must be exactly 4000 m^3.

   (a) Find and sketch the domain of the heat loss as a function of the lengths of the sides.

   (b) Find the dimensions that minimize the heat loss, checking both the critical points and the boundary.

   (c) Could you design a building with even less heat loss if the restrictions on the lengths of the walls were removed?

4. (14.8 #21) Find the extreme values of $f(x, y) = e^{-xy}$ on the domain $x^2 + 4y^2 \leq 1$.

5. (14.8 #37). Use Lagrange multipliers to provide an alternate solution to problem 2.