Math 11, Multivariable Calculus
Written Homework 3

1. Section 14.3: Problem 90. The wind chill index is modeled by the function

$$W = 13.12 + 0.6215T - 11.37v^{0.16} + 0.3965Tv^{0.16},$$

where T is temperature (°C), and v is the wind speed in km/h. When $T = -15$°C and $v = 30$ km/h, by how much do you expect the apparent temperature W to drop if the actual temperature decreases by 1°C? What if the wind speed increases by 1 km/h?

2. Section 14.4: Problem 38. The pressure, volume, and temperature of a mole of an ideal gas are related by the equation $PV = 8.31T$, where P is measured in kilopascals, V in liters, and T in °K. Use differentials to estimate the approximate change in the pressure if the volume increases from 12 to 12.3 liters and the temperature decreases from 310°K to 305°K.

3. Section 14.4: Problem 42. Suppose that we need the equation of the tangent plane to a surface S at the point $P = (2, 1, 3)$, but we know only that the curves $\mathbf{r}_1(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \rangle$, and $r_2(u) = \langle 1 + u^2, 2u^3 - 1, 2u + 1 \rangle$ lie on the surface and pass through P. Find the equation of the tangent plane at P.

4. Section 14.5: Problem 36. Wheat production in a given year depends on the average temperature T and the annual rainfall R. Scientists estimate that the average temperature is rising at a rate of 0.15°C/year and rainfall is decreasing at a rate of 0.1 cm/year. They also estimate that at current production levels, $\dfrac{\partial W}{\partial T} = -2$ and $\dfrac{\partial W}{\partial R} = 8$.
   (a) What is the significance of the signs of these partial derivatives?
   (b) Estimate the current rate of change of wheat production dW/dt.

5. Section 14.6: Problem 34. Suppose that you are hiking a hill whose shape is given by the equation $z = f(x, y) = 1000 - 0.005x^2 - 0.01y^2$ where x, y and z are measured in meters, and you are standing at a point with coordinates $(60, 40, 966)$. The positive x axis points east and the positive y axis points north.
   (a) If you walk due south, will you start to ascend or descend? At what rate?
   (b) If you walk northwest, will you start to ascend or descend? At what rate?
   (c) In which direction is the slope largest? What is the rate of ascent in that direction? At what angle above the horizontal does the path in that direction begin?

6. Section 14.6: Problem 49. If $f(x, y) = xy$, find the gradient vector $\nabla f(3, 2)$ and use it to find the tangent line to the level curve $f(x, y) = 6$ at the point $(3, 2)$. Sketch the tangent line, level curve and gradient vector.