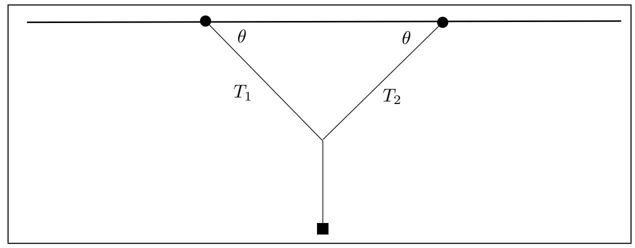
Math 11, Multivariable Calculus
Written Homework 1

1. Consider a variation on Example 7 from section 12.2. Suppose we have a 10-pound weight suspended from two wires as in the diagram below, and that the wires make the same angle with the supporting structure. One goal of the problem is to compute the resulting tensions ($\mathbf{T}_j$) as a function of θ. But before we start the computation, it is instructive to tune our expectations. Instead of anchor points, suppose each of these was a scale which measured the magnitude of the tension (like a scale in the produce section of a food market). Just what are our expectations? If $\theta = \pi/2$ (meaning the two anchor points were on top of each other), we would no doubt expect each scale to read 5 pounds. Presumably, as θ changes value, so should the resulting tensions. But consider: do you think there is a value of θ where *each* scale reads 10 pounds? Could there be a value of θ where the scales read *more* than 10 pounds? Ok, let's find out.



   (a) Using Example 7 in the text for guidance if needed, write expressions for the tensions $\mathbf{T}_j$ in terms of their lengths $|\mathbf{T}_j|$ and the angle θ.

   (b) Compute the magnitude of the tensions: $|\mathbf{T}_j|$, $j = 1, 2$ as a function of θ.

   (c) Compute the magnitudes for $\theta = \pi/3, \pi/4, \pi/6$. How do these values compare to your predictions above? Do these numbers make intuitive sense?

2. section 12.2: #39. A boatman wants to cross a canal that is 3km wide and wants to land at a point 2km upstream from his starting point. The current in the canal flows at 3.5km/h and the speed of the boat is 13 km/h.

   (a) In what direction should he steer?
   (b) How long will the trip take?

3. section 12.3: #64. Show that if $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ are orthogonal, then the vectors $\mathbf{u}$ and $\mathbf{v}$ have the same length.

4. section 12.4: #53. Suppose that $\mathbf{a} \neq \mathbf{0}$.

   (a) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?
   (b) If $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?
   (c) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ and $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?

5. section 12.5: #78. Find the distance between the skew lines with parametric equations $x = 1 + t$, $y = 1 + 6t$, $z = 2t$, and $x = 1 + 2s$, $y = 5 + 15s$, $z = -2 + 6s$.