V63.0123-1 : Calculus III. Sample Midterm2

You have 60 minutes. Non-graphing calculators and a single side of letter paper equations are allowed.

1. [12 points]

   Let $f(x, y) = \frac{x}{y} + \frac{y}{x}$.

   (a) What is the domain of f?

   (b) Compute ∇f at a general point (x, y).

   (c) Show that $\nabla f(x, y)$ is always perpendicular to $\mathbf{r} = (x, y)$.

   (d) Does $\lim_{(x,y)\to(0,0)} f(x, y)$ exist? If so, compute the limit. If not, explain why.

2. [10 points]

   Find the integral of $f(x, y, z) = x$ over the three-dimensional domain bounded by the planes $z = 0$ and $x + z = 1$ and the parabolic cylinder $x = y^2$.

3. [8 points]

   Find the x coordinate of the center of mass of a lamina occupying the region $D = \{(x, y) | x^2 + y^2 \leq 1, x \geq 0\}$ with constant density.

4. [10 points]

   If $f = xy + yz + zx$, with $x = st$, $y = e^{st}$ and $z = t^2$, find $\partial f/\partial s$ and $\partial f/\partial t$ at the coordinates $(s, t) = (0, 1)$.

5. [10 points]

   Find and categorize the critical points of $f(x, y) = x^2y + y^2 - y$. Find the value of f at any local extrema. Use what you have found to help you sketch a contour plot of $f(x, y)$, with the critical points shown. [Hint: you may want to plot the contour line(s) $f = 0$ first].

Practise problems from Stewart book (note exam may also have non-Stewart style problems, but will be mainly Stewart-style). Also make sure you understand HW4–8.

15.1: 9, 27, 29, 31, 39.
15.2: 7, 9.
15.3: 25, 45, 47, 53, 67, 79.
15.4: 3, 13, 17.
15.5: 9, 11, 19, 43.
15.6: 21, 23, 25, 29, 35.
15.7: 7, 9, 33, 37, 47.
15.8: 5, 9, 19.
15 Review (p.994): Concept check 1, 6, 7, 15, 17.

16.1: 13.
16.2: 17, 29.
16.3: 7, 11, 17, 27.
16.4: 15, 21, 23, 31.
16.5: 3, 5, 11.
16.6: 7, 9, 19.
16.7: 7, 19, 45.
16.8: 13, 19, 23.
16 Review (p.1068): Concept check 2a–c, 3, 6.