Math 11 Fall 2004

Multivariable Calculus

for Two-Term Advanced Placement First-Year Students

First Midterm Exam

Wednesday October 20, 3:30-4:45 PM

Your name (please print): ______________________________

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted**. You are allowed to bring one $A4$ size sheet of paper with any data you want written on it. You must justify all of your answers to receive credit, unless instructed otherwise in a given problem. On the multiple choice questions, only the answer you mark on the scantron form will be counted and justifications can be minimal.

You have **one hour and fifteen minutes** to work on all **15** problems. Please do all your work in this exam booklet.

The Honor Principle requires that you neither give nor receive any aid on this exam.

Grader's use only

1-10: Multiple Choice: ________ /50

11. ______________ /10

12. ______________ /10

13. ______________ /10

14. ______________ /10

15. ______________ /10

Total: ________ /100

(1) Let $\vec{\mathbf{v}} = \langle 1, 2, 4 \rangle$ and $\vec{\mathbf{u}} = \langle 2, 3, \sqrt{8} \rangle$. Which one of the following statements is true.

(a): the length of $\vec{\mathbf{v}}$ is 23,
(b): $\vec{\mathbf{u}}$ is shorter than $\vec{\mathbf{v}}$,
(c): $\vec{\mathbf{v}}$ is shorter than $\vec{\mathbf{u}}$,
(d): $\vec{\mathbf{v}}$ and $\vec{\mathbf{u}}$ have equal lengths.

(2) Let $\vec{\mathbf{v}} = \langle 1, 2, 0 \rangle$ and $\vec{\mathbf{u}} = \langle -2, 1, 3 \rangle$. Then the angle between the vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{u}}$ is:

(a): $\frac{\pi}{3}$,
(b): 0,
(c): π,
(d): $\frac{\pi}{2}$.

(3) Let $\vec{\mathbf{r}}(t) = \langle x(t), y(t), z(t)\rangle$, $1 \le t \le 3$. Which one of the following statements is true (only one of the statements is true):

(a): $\int_1^3 \vec{\mathbf{r}}'(t)dt$ is the distance between $\vec{\mathbf{r}}(1)$ and $\vec{\mathbf{r}}(3)$;
(b): $\left|\int_1^3 \vec{\mathbf{r}}'(t)dt\right|$ is the length of the curve $\vec{\mathbf{r}}(t)$ between $\vec{\mathbf{r}}(1)$ and $\vec{\mathbf{r}}(3)$;
(c): $\int_1^3 \left|\vec{\mathbf{r}}'(t)\right|dt$ is the distance between $\vec{\mathbf{r}}(1)$ and $\vec{\mathbf{r}}(3)$;
(d): $\int_1^3 \vec{\mathbf{r}}'(t)dt$ is the vector from $\vec{\mathbf{r}}(1)$ to $\vec{\mathbf{r}}(3)$.

(4) The

$$\lim_{t\to 0}\langle \frac{\sin 3t}{t}, \frac{e^t - 1}{t}, \frac{t^3 + t^2}{t^5 + 2t^2}\rangle =$$

(a): $\langle 1, 1, 1\rangle$,
(b): $\langle 3, 1, \frac{1}{2}\rangle$,
(c): $\langle 3, 1, 1\rangle$,
(d): $\langle 1, 1, \frac{1}{2}\rangle$

(5) State whether the following are true or false for all the three-dimensional vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ in the order (1), (2), (3).

(1): $(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{c}} = (\vec{\mathbf{b}} \cdot \vec{\mathbf{a}})\vec{\mathbf{c}}$;
(2): $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}} = (\vec{\mathbf{b}} \times \vec{\mathbf{a}}) \times \vec{\mathbf{c}}$;
(3): $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \cdot \vec{\mathbf{a}} = 0$.

(a): TFF
(b): TFT
(c): TTF
(d): TTT

(6) Consider the three planes given by the equations

$$P_1 : x + 3y - z + 10 = 0,$$
$$P_2 : 2x + 6y = 17 - 2z;$$
$$P_3 : 3x + 3z = 5.$$

Which one of the following statements is true? (Only one of the statements is true.)

(a): none of these planes are perpendicular to each other;
(b): two of these planes are perpendicular to each other and two of the planes are parallel to each other;
(c): two of the planes are perpendicular to each other and none of the planes are parallel to each other;
(d): two of the planes are parallel to each other and non of the planes are perpendicular to each other.

(7) Let $\vec{\mathbf{c}}(t) = (t^2, -2t+3, 2\sin t)$ be a curve. Which one of the following is the equation of the tangent line to $\vec{\mathbf{c}}(t)$ at the point $\vec{\mathbf{c}}(1)$?

(a): $\vec{\mathbf{r}}(t) = \langle 1, 1, 2\sin 1\rangle + t\langle 2, -2, 2\sin 1\rangle$;
(b): $\vec{\mathbf{r}}(t) = \langle -1, -1, 2\sin 1\rangle + t\langle 2, -2, 2\cos 1\rangle$;
(c): $x = 1 - t,\ y = 1 + t,\ z = 2\sin 1 - t(\cos 1)$;
(d): $\langle 2, -2, 2\cos 1\rangle \cdot \big(\langle x, y, z\rangle - \langle 1, 1, 2\sin 1\rangle\big) = 0$.

(8) Let $f(x, y)$ be a differentiable function of two variables. Let $x = g(t), y = h(t)$ where $g(t)$ and $h(t)$ are also differentiable functions. Find $\frac{df}{dt}$ at $t = 1$, provided that $g(1) = 3,\ h(1) = 5,\ g'(1) = 4,\ h'(1) = 1,\ f_x(3, 5) = 1,\ f_y(3, 5) = 3,\ f_x(5, 3) = 4,$ $f_y(5, 3) = 7$. **(You will not need some of this data.)** Answer is
$\frac{df}{dt}(1) =$

(a): 23,
(b): 11,
(c): 4,
(d): 7.

(9) Let $f(x, y, z) = x^2 + yx + e^z$. Then the function $f(x, y, z)$ at the point $P(1, 1, 0)$ **decreases** most rapidly in the direction of:

(a): $\langle 0, \frac{3}{5}, \frac{4}{5} \rangle$
(b): $\langle 1, 1, 0 \rangle$
(c): $\langle -3, -1, -1 \rangle$
(d): $\langle 3, 1, 1 \rangle$

(10) Let $f(x, y) = x^2 + 3y^3 - 2x - y + 17$. Then the point $(1, \frac{1}{3})$ is

(a): not a critical point;
(b): is a saddle point;
(c): is a local minimum point;
(d): is a local maximum point.

(11) Find the directional derivative of $f(x, y, z) = (x^2 + y^3 + z^4)$ in the direction of the vector $\langle 1, 2, 3 \rangle$ at the point $P(1, -1, 0)$.

(12) Find the trajectory $\vec{\mathbf{r}}(t)$ of a point moving with acceleration $e^t\vec{\mathbf{k}}$ provided that the initial velocity $\vec{\mathbf{v}}(0) = 2\vec{\mathbf{i}} + \vec{\mathbf{k}}$ and the initial position $\vec{\mathbf{r}}(0) = \vec{\mathbf{i}} + 3\vec{\mathbf{j}} + \vec{\mathbf{k}}$.

(13) Find the volume of the parallelepiped determined by the vectors

$$\vec{\mathbf{a}} = \langle 0, 1, 0 \rangle,$$
$$\vec{\mathbf{b}} = \langle 0, 1, 1 \rangle,$$
$$\vec{\mathbf{c}} = \langle 3, 1, 0 \rangle.$$

(14) Find the length of the curve $\vec{\mathbf{r}}(t) = (\frac{1}{2}t^2, \ln t, t\sqrt{2})$, for $1 \leq t \leq 3$.

(15) Find the parametric equation of the line through $P(1, 2, 3)$ that is perpendicular to the plane $x + y + z = 17$.