Hint for Problem 52-e

Using d for down and u for up, we could have $uudduuddudud$ as our Catalan path. Suppose that $i = 5$. The fifth upstep is the u in position 9. Thus $F = uudduudd$, $U = u$, and $B = dud$. Now BUF is $duduuudduudd$. This is a Dyck path that begins by going below the x-axis. The $d's$ in positions 1 and 3 take the path to the y-coordinate -1. Then the y coordinate climbs to 2, goes back to 0, up to 2 again, and finally down to 0. So the absolute minimum is -1, and it occurs in the first and third position. There are five u's after the third positon. So this Dyck path is in the block B_5 of our partition. Now comes the crucial question. Why were there five u's after that last absolute minimum in position 3? Try with the same path and $i = 3$. Figure out why there are three u's after the last absolute minimum in the resulting path. All this discussion should explain why when $i = 5$, the set of all Catalan paths is mapped into the set B_5. Keeping $i = 5$ for a while, try to see why this correspondence between Catalan paths and B_5 is a bijection. Then, if you need to, do the same thing with $i = 3$. This should give you enough insight to do the general case.