

Hint for Problem 224-a

A Catalan path could touch the x -axis several times before it reaches $(2n, 0)$. Its first touch can be any point $(2i, 0)$ between $(2, 0)$ and $(2n, 0)$. For the path to touch first at $(2i, 0)$, the path must start with an upstep and then proceed as a Dyck path from $(1, 1)$ to $(2i - 1, 1)$. From there it must take a downstep. Can you see a bijection between such Dyck paths and Catalan paths of a certain kind?