Hint for Problem 204

If infinitely many of the polynomials had a nonzero coefficient for q, would the product make any sense.