

Hint for Problem 126

Note that $\binom{n+k-1}{k} = \binom{n+k-1}{n-1}$. So we have to figure out how choosing either k elements or $n-1$ elements out of $n+k-1$ elements constitutes the choice of a multiset. We really have no idea what set of $n+k-1$ objects to use, so why not use $[n+k-1]$? If we choose $n-1$ of these objects, there are k left over, the same number as the number of elements of our multiset. Since our multiset is supposed to be chosen from an n -element set, perhaps we should let the n -element set be $[n]$. From our choice of $n-1$ numbers, we have to decide on the multiplicity of 1 through n . For example with $n=4$ and $k=6$, we have $n+k-1=9$. Here, shown with underlines, is a selection of $3=n-1$ elements from $[9]$: 1, 2, 3, 4, 5, 6, 7, 8, 9. Note that $\binom{n+k-1}{k} = \binom{n+k-1}{n-1}$. How do the underlined elements give us a multiset of size 6 chosen from an $[4]$ -element set? In this case, 1 has multiplicity 2, 2 has multiplicity 1, 3 has multiplicity 2, and 4 has multiplicity 1.