Workshop on Teaching Introductory Combinatorics by Guided Discovery Dartmouth College, Hanover, New Hampshire

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Leaders: Ken Bogart, Dartmouth College and Karen Collins, Wesleyan University

Combinatorics is an ideal subject for students who are learning to think mathematically. Many of its questions are easy to understand and yet are an appropriate challenge for a student early in a mathematics major. In introductory problems, any proofs one may ask for are simply written justifications for the assertion the student is making. They often mirror computations students make in problem solving. This makes the subject ideal for a student who is near the beginning of his or her mathematical career. Nonetheless, it doesn't take too much introduction to bring all students to problems that have intellectual meat, so the subject is a good place for a group of students of mixed abilities and backgrounds. The main theorems of combinatorics can be motivated by problems that lead a student to discover the statements of the theorems. This makes it possible to present combinatorics to students as a collection of problems to solve, with definitions in appropriate places and some summarizing discussions either written among the problems or done in class (or both). Since the problems guide the students to discover the subject, we call the process of learning from such a problem set "guided discovery." The NSF has funded the development of a set of notes for teaching introductory combinatorics by guided discovery.¹

Teaching by Guided Discovery

Guided discovery nurtures students' mental processs of constructing their own understanding of a subject. Learning theory suggests that students learn the most when they have to grapple with and reformulate their understanding of a subject. While some students thrive on this process on their own, many students find that group brainstorming sessions ease the process. Such sessions help them to recognize that others struggle as they do. Further, group brainstroming sessions bring several minds to bear on a problem, thus increasing the chances that a fruitful idea will come up. Even when a student first gets an idea from another student, she or he sees the idea in the context of intense thought about a problem. Usually the idea is only partly formed when it comes up in a brainstorming session, so the students must complete it either as a group or on their own. Thus students have the potential to learn more deeply from an idea they first encounter in a group brainstorming session than from one they first encounter in a lecture. Guided discovery teaching through using such groups is called guided group discovery. In guided group discovery teaching, one role of the teacher is to monitor some of these brainstroming sessions to help students work together

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in productive ways. But a group brainstorming session is unlikely to help a student remember an idea for more than a short time. In writing up a full solution to the problem on his or her own, a student must reflect deeply on the idea, thinking about how, exactly, it solves the problem. This helps the student get the idea into a broader context in longer term memory. Thus in guided group discovery teaching we require each student to write up all the significant results that are found in the problems. As the instructor reads these solutions, he or she often finds that the student either has not constructed a sufficiently robust mental image of the concepts involved in the problem to really solve the problem, that the student is bringing a significant misconception to his or her solution of the problem, or that the student appears to understand a solution to the problem but cannot yet explain in written words what he or she is thinking. Thus another role for the instructor in guided discovery teaching is to give the students regular feedback about their written work and encourage (or require) the students to rewrite their work until it is both correct and coherent. Still another role for the teacher is to help students discover the "big picture" through discussions of relationships among the problems and the main themes of the subject the students are developing.

The workshop.

In this workshop, we will focus on teaching introductory combinatorics by guided group discovery. The workshop will feature opportunities for the participants to try their hand at learning by guided group discovery in an unfamiliar area. It will provide opportunities for reading and discussion of selected material from mathematics education research relevant to guided discovery teaching. Participants will be able to work with the notes mentioned earlier to create lesson plans suitable for their students. There will be a "workshop within the workshop" on plannning, preparing, and submitting proposals to the National Science Foundation for "adapt and adopt" grants to use these notes or for "course and curriculum development" grants to create guided discovery materials for other courses.

Recreational opportunities.

In the immediate Hanover area there are opportunities for canoeing on and swimming in the Connecticut River (rental canoes are available), biking on back roads or trails, and hiking on the Appalachian Trail. If weather permits, an afternoon will be devoted to a recreational outing.

Travel to the workshop.

The workshop will be held on the Dartmouth campus in Hanover, NH. From the Northeast, Hanover is most accessible by auto. The Lebanon Regional Airport (LEB), about seven miles away has several daily US Air commuter flights. The airport in Manchester, NH is about 80 miles away and is served by many major carriers and auto rental companies. Boston's Logan Airport is about 135 miles

away; regular service from Logan Airport to the Dartmouth campus is provided by Dartmouth Coach, a bus service.

Priorities for accepting applicants for the workshop.

Faculty and graduate students in all areas of the mathematical sciences are welcome to apply to participate, as are mathematically adventurous school teachers. First priority will be given to faculty members in departments in mathematics or related fields whose chairs can provide them with reasonable assurance that they will have an opportunity to try guided discovery teaching in combinatorics sometime in the next several years. Second priority will be given to faculty who do not often have professional development opportunities and who would like to explore the idea of incorporating guided discovery into their teaching in mathematics and related fields. These priorities will guide the selection of applicants until Feb. 15, 2003. After Feb. 15, 2003 applications will be processed on a first-come, first-served basis. The critera for accepting applicants will be the potential value of the workshop to the applicant, the applicant's potential for helping the workshop reach a broad cross section of the American higher education system, and the applicant's potential for contributing to a working group of people with similar interests within the workshop.

Financial support.

NSF policy is that participant's home institutions should fund their travel to and from the workshop. Thus workshop participants will be responsible for their own travel to and from the workshop. Workshop organizers will be happy to write a polite letter to an appropriate administrative person at a participant's home institution explaining the NSF policy and asking that the institution attempt to provide travel funds. Funding from the National Science Foundation will provide single occupancy dormitory rooms (with bathrooms shared by a suite of rooms) for five nights, two meals a day at Dartmouth College facilities, and a modest allowance towards the actual costs of one meal daily either at Dartmouth facilities or local restaurants (alcoholic beverages **not** included), both for five days. Requests for additional support to allow a Saturday night stayover will be considered on a funds available basis.

Application forms.

Application forms may be downloaded at

www.math.dartmouth.edu/kpbbogart/comboworkshop.html