Teaching Combinatorics Through Guided Discovery:

a summary evaluation Report

submitted to the principal investigator ,

the advisory committee,

and the

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> The obvious difference [between guided discovery combinatorics and other math classes] was that it was entirely the "active learning" setting in the classroom. [The instructor] rarely lectured to us, but rather we did the problems in small groups and explored the ideas and concepts on our own. Once the problems were complete, we would pause and discuss them as a class to go over and [correct] confusion, misconceptions, etc... The class was fun, and challenging. I enjoyed going and I really felt a responsibility to my group and the instructor, especially when comparing this class to a lecture-based class where I could remain nameless and unnoticed for the entire semester.....Compared to the combinatorics class that I had taken previously, the knowledge I gained in this class was far greater. Rather than simply being presented with a formula, often I derived it myself and therefore I had a much deeper understanding (and appreciation!) for it. This course really solidified my understanding of math ... it was fun and exciting to explore and discover things for myself. I really think that this method of teaching is much more engaging, gratifying and motivating for a student.! *[Guided Discovery Student]*

We interviewed 21 of the 28 students enrolled in "Combinatorics by Guided Discovery" at two institutions, and 85% agreed with the student above that they learned more mathematics, and learned it more deeply, by the guided discovery method. They also came to a better understanding of how mathematics is done. Many learned, as this student said, "to be more creative in how I solved problems," others that "there are usually many, many ways to solve a problem." As one summarized, "[The course] opened my eyes to the way math is done—collaboration, being creative, exploring the stuff, not worrying that there is no formula because you are the one who is supposed to be discovering it." There were a few students for whom the course was too easy, a few who found it too challenging, and a few who resented what they termed having to "teach themselves," rather than be "taught" by a lecture. But the large majority were liberated and empowered. This report describes the course, its outcomes for student learning, and the persistent pedagogical challenges it presents.

THE PROJECT

The guided discovery technique piloted in this project conjoins two strategies—self-paced learning and group work—in an effort to make learning more efficient for all students. The carefully structured problem sequence in guided discovery is designed to carry students step by manageable step through the material, so that they construct the mathematical concepts in a cyclical "bottom up" approach, instead of receiving it in the "top down" textbook method (and then hoping to deconstruct and understand by solving problems). We know that students learn and retain more when they move forward from a strong base of understanding and suffer when they advance prematurely. In guided discovery, students move at their own pace, going forward only when their foundation is secure. A grading system that rewards mastery before moving to the next level reinforces the message that genuine understanding is the goal. Working in small groups reduces the isolation math students often feel and increases the brain power available for problem solving. Students feed off one another's energy and ideas to move more productively and happily through the material. In addition to becoming aware of multiple perspectives, students learn from peers' more accessible explanations and deepen their own understanding by explaining to others. Frequent close contact with the instructor offers a window on how mathematicians think, especially when novel issues are raised. This project hypothesized that students would learn combinatorics better and understand more deeply using guided discovery than through a standard lecture presentation. It further sought to identify those pedagogical strategies involved in guided discovery that promote learning. The project evaluation consequently focussed on comparing how well students learned the same material in the two modes and identifying successful pedagogical strategies.

EVALUATION DESIGN. We compared learning by documenting student accomplishment and experience at two institutions (one large, one small) where an instructor first taught combinatorics by the lecture method and then, in a subsequent term, by guided discovery. One instructor was female, the other male. Piloting the course at different kinds of institutions allowed us to gauge the effectiveness of guided discovery with students of different interests and backgrounds. While the heterogeneous student populations provided a good test of the method's flexibility, it must be remembered that the student outcomes reported here also reflect the math preparation students brought to the course. The only valid comparisons are within institutions, not between them.

The population of combinatorics students is small, and no single measure will yield unequivocal results, so we employed multiple measures, both quantitative and qualitative, to substantiate our conclusions. To judge the how well and deeply students understand the material, near the conclusion of the course students participated in friendly oral examinations about combinatorics with an outside combinatorialist. He scored their knowledge on a 100-point scale, using both the content of their responses and a more subtle index—their use of mathematical language—to gauge the depth of their of understanding. These hour-long conversations with a mathematician in the field allowed us to assess student learning with an accuracy, and at a depth, rarely afforded to experimental curricula. We also asked students to rate their own learning using a self-assessment instrument. Because a successful mathematics course ought to also improve students' thinking and problem solving skills and deepen their understanding of mathematics as an activity, we used a pre-post survey to measure changes in students' attitudes about mathematics. Interviews documented the students' perspective on pedagogical strategies, while classroom observation by the evaluator, including observation of the "alpha" iteration of guided discovery not included in the comparative design, provided an independent record of instruction strategies and classroom activity to contextualize student data. The research design is schematized below, with the number of students represented in each assessment activity shown in parentheses.

TABLE 1. EVALUATION DESIGN

	LECTURE	GUIDED DISCOVERY
INSTITUTION A •*Small , under-graduate college* •*Female instructor*	*Student enrollment = 11* *2/3 math majors* • observation x 5 • pre-post survey (7) • learning self-assessment (9) • content interviews (5) • pedagogical interviews (5)	*Student enrollment = 16* *9/10 math majors, 2 grad students* • observation x 10 • pre-post survey (15) • learning self-assessment (15) • content interviews (9) • pedagogical interviews (11)
INSTITUTION B •*Large university*	*Student enrollment =20* *2/3 math majors* • pre-post survey (13)	*Student enrollment =12* *>1/2 math majors, 2/3 grad st.* • pre-post survey (11)

• *Male instructor*	• learning self-assessment (16) • content interviews (9) • email pedagogical interviews (5)	• learning self-assessment (11) • content interviews (12) • email pedagogical interviews (10)

STUDENT LEARNING OUTCOMES. On most measures, guided discovery students performed as well as, and often better than, lecture students. Ratings by the external combinatorialist and students' own evaluation of their progress showed that overall, students at both schools learned as well or better through guided discovery than through the traditional lecture mode. There were, however, important gender differences. We include a breakdown by gender for all evaluation measures, despite the fact that the number of women in some classes is very small, and the inferential power of the data weak. If we look forward to the time when there are more women students and instructors, it is important to understand, as best we can with the limited data presently available, how the effectiveness of pedagogical strategies varies according to the teacher/student gender configuration. In this experiment, the greatest gains were made by students of the opposite sex of the instructor: men at Institution A and women at Institution B. Students who appear to benefit least are those who are probably privileged in the lecture mode: men with a male instructor. Thus although male students with a male instructor performed best overall in both modes, they benefited least by the switch to guided discovery.

Table 2 summarizes these results. by showing on which measures guided discovery students outperformed lecture students.

TABLE 2. DID GUIDED DISCOVERY LEAD TO IMPROVEMENT?

	All Improved		Men Improved		Women Improved	
Measure	Institution A	Institution B	Institution A	Institution B	Institution A	Institution B
Content Intv.	yes	no change	yes	no	yes	yes
Self Assess	No change	yes	yes	yes	no	yes
Survey	yes	no	yes	no	yes	yes

Another way to summarize findings is to compare students' performance, categorized by their gender and that of their instructor, across all measures. We created a rough index of overall outcomes by treating all the measures as percentages and summing them.[1] As Table 3 shows, the rank order of performance does not change across teaching modes, but the range is notably reduced. Students of both sexes at Institution A, with a female instructor, performed better in guided discovery. At Institution B, with a male instructor, men's scores held relatively steady while women's scores improved to almost the level of men's.

[1] This requires assuming, among other things, that all three measures are of equal importance. This may or may not be so; that is why this is a rough index. The content interview was "graded" on a scale of 100. The learning self-assessment index was created by expressing each student's total score as a percentage of the perfect total score. The survey index was created by dividing each student's post-score by the pre-score and multiplying by 100.

TABLE 3. PERFORMANCE SCORES BY INSTRUCTOR AND STUDENT GENDER

	LECTURE				GUIDED DISCOVERY			
INSTITUTION (instructor)	Content	SA	Survey	Index	Content	SA	Survey	Index
B (M) all	88	70	106	264	88	78	99	265
A (F) all	83	64	95	242	89	64	102	255
B (M) males	92	72	109	273	89	78	100	267
B (M) females	82	64	95	241	87	78	96	261
A (F) males	85	54	96	235	91	64	102	257
A (F) females	75	70	90	235	87	62	102	251

These findings raise questions about the complex gender interactions between teacher and student. Does the lecture mode favor the student of the same sex as the lecturer, perhaps by encoding subtle gender-specific cues in verbal and body language (and perhaps in other ways)? However lecture effects might ultimately be explained, why would guided discovery counteract them? Perhaps when instructors try something new, they tend to put aside familiar modes of interaction and speech. Or perhaps, because instructors spend much of their time interacting with individuals or small groups, they tailor explanations and language to the individual questioner rather than a presumed majority or "ideal listener." The findings do suggest, albeit tentatively, that guided discovery may neutralize gender bias in teaching, increasing the efficiency of instructors with opposite-sex students without significantly diminishing their effectiveness with same-sex students.

Components of Success; Challenges for Improvement

Each of the three courses (one alpha, two betas) can justifiably be called a "success," but each implemented the method somewhat differently. Comparing student responses to these three variants allows us to identify those strategies that seemed most conducive to learning. We ask the same question of each version (e.g., how were groups formed? how was class time structured?), then turn to student feedback to extract those methods that seem most successful. Not surprisingly, the same new elements that made the course interesting and fun were also the most challenging to orchestrate. Three issues stand out: pacing, forming and managing groups, and providing intellectual structure.

SETTING THE PACE THROUGH A STRUCTURED SEQUENCE OF PROBLEMS. Pacing is an unacknowledged issue in almost all mathematics courses, and guided discovery is to be credited for acknowledging the issue and making its solution a stated goal. We know that students do not maintain a consistent pace throughout a term. Compared to a lecture format, where everybody is on the same page but reading at different levels of comprehension, in guided discovery students are often on different pages, but at very similar levels of comprehension. Even allowing for variable progress through the term, some students will ultimately proceed faster through the material than others, raising obvious classroom management problems. We do not want to restrain those eager to challenge themselves, nor do we want to push students beyond their comprehension. But there is a limit to how wide the gap between the fastest and slowest learners can become before the classroom devolves into a venue for so many small group tutorials, sacrificing the intellectual and social advantages of classroom discussion and preventing the kind of recap and review that helps students construct the big picture.

The challenge for guided discovery is to keep everyone progressing more or less together without stifling the faster students' enthusiasm. Well-functioning groups can accelerate the rate of learning for many students who are having difficulty, and some who could have moved faster were satisfied by learning more deeply. But some students were bored, and disgruntled: "I really didn't feel like I was challenged in this course. Its tough to stay engaged when you are not feeling challenged." Some instructors developed strategies to address pacing issues; students proposed additional ones.

One piloted strategy set the number of problems required to receive full homework credit at less than 100%. Students who wanted to cover all the topics chose fewer problems for each topic and moved faster through the material; those in the other groups spent more time, and worked more problems, for each topic, but did not complete all topics. Tests covered only material familiar to all. A second approach differentiates students by the depth, not extent, of their exposure to the material, dividing problems for each topic into essential "core" problems required of all students and ancillary problems, which offer more variations (and challenges) on the theme. Other possibilities: advanced students might complete special projects, give presentations to the class, or become tutors in topics they especially enjoy. These ideas are suggestive, models to think about rather than an exhaustive list.

In all of them, however, grading considerations are critical. Most students told us they would do (or actually did) extra work just for the pleasure of accomplishment, and no students said they would be embarrassed (or even reluctant) to be identified as a member of a slower group if going more slowly would help them learn better. But they will not proceed more slowly if it would jeopardize their grade. The grading scheme in guided discovery is heavily weighted toward homework (usually 45% to 50% of the total grade), and students may resubmit problems until they are correct. This policy sends the clear message that deep understanding is valued above speed. Almost all students endorsed this policy, and rejected any alteration which would deny a respectable grade to those who commit to deep learning. As this student said,

> If I [hadn't] completed all the problems in the book, I think I would have gotten a lot more understanding out of the class. There's nothing wrong with going slow as long as you understand it. [But] if I had done that and only completed like 75% of the problems, that would have completely killed my grade.

FORMING AND MANAGING GROUPS. Working on mathematics in small groups lies at the heart of the guided discovery method, but for group work to be successful the problems must be hard enough that students can't solve them easily on their own, students must be comfortable in the group, and they must feel accountable to their group, that is, everybody must give their best effort. None of these things happens automatically when students break into groups. Forming and managing groups is a major—and typically unfamiliar—task for the instructor. Superintending social relationships in the classroom is not extraneous; it is critical to successful group function, which in turn is central to the success of the course. Here is what we have learned about successful group functioning.

Groups work well when members are all at the same general level of competence. Unlike the differentiated tasks of the typical K-12 group, the goal of guided discovery groups is to harness more brainpower for the problem. Brainstorming is most effective when everyone can grasp new ideas quickly and build on them. But group homogeneity is also important for social reasons. No one likes always to be giving help (the bossy know-it-all) or always to be receiving help (the dumb one). When groups lose their egalitarian ambiance, a social discomfort settles in that affects the intellectual functioning of the group. Our experience suggest that groups should include both sexes and be comprised of three or more students to get a diversity of perspectives. It is the instructor's responsibility to see that students (unless the seriously object to group work) are at all times in a group where they are comfortable socially and at the right level. This may change during the term, so constant monitoring is required.

Groups work well when students feel that they are friends. Groups only move forward efficiently when participants can speak frankly—critiquing ideas and revealing their own weaknesses. Students are unwilling to do either among people whose responses they cannot predict (i.e., those they don't know), who might respond angrily to perceived criticism or judgmentally to revealed weakness. The rough and tumble of intellectual debate needs to be cushioned by the social caring and respect that is achieved through familiarity. It is up to the instructor to establish an atmosphere that promotes easy give and take. Efforts to break down social isolation in the class, even by so simple a measure as calling students by name, pay dividends in terms of class discussion and group work effectiveness.

Groups work well when the problem is at the right level, neither so hard that no one can get a purchase on it, nor so easy that the solution is quickly evident to most. Given the overhead in organization and social accommodation, group work only makes sense when its value added is greater than working alone. Students' biggest complaint about group work concerned being "stuck" or "stalled," lacking the insight to take the first (or next) step. During in-class work that situation was quickly remedied by a well-placed cue from the professor, and it's important that the instructor visit each group frequently during the class, even if they don't appear to be in distress.

Groups work well when everybody tries hard. Even in groups where some students had more resources than others and the balance of giving and taking may have been somewhat uneven, the group functioned well if everyone worked to their ability. What was not forgiven was profiting from others work without putting in ones own full effort. Several students with weaker backgrounds pointed out how valuable it was to have in their group a math major, who brought a richer repertoire of problem attack skills to the table. There was no evidence that these math majors resented being group leaders, so long as others gave their best effort.

IMPOSING INTELLECTUAL STRUCTURE. When asked what would make the course better, students in all three courses answered with one voice: the problems need to be placed in an intellectual framework, and this needs to be done in class. Whether they asked for "periodic updates," "more lecture," "introduction to concepts," "guideposts," "explanations," "more instruction," "review," or "pulling it all together," the message was the same. As one student said, ." I feel like there should be some structure...going over problems on the board, summarizing: okay this is what we have done so far, any questions about this, this is how this fits to this.."

Graduate students missed the "deeper mathematics" they might have accessed more easily with the help of a lecture, while undergraduates (especially the women) felt more lecture would help them understand the concepts. But they were both explicit that they weren't asking for "old school" lectures, but for short (ten to fifteen minute) explanations of where they had been, where they were going, how those two were related, and why they were doing what they were doing. When they had reached a plateau, or developed an important idea, they wanted to know that a milestone had been achieved. And if it had a name, they wanted to know that also. Several worried that they lacked the standard mathematical language for their own knowledge. As a student in the alpha course said, "If someone asked me if I knew such and such, I might, but I might not know it. I would only know that I can do problem 169." They were also explicit that they did not believe providing such intellectual scaffolding would compromise the benefits of working through the material for oneself.

CONCLUSIONS

Student results suggest that most students learn more, and learn more deeply, with the guided discovery method. Interviews confirm that "being forced to discover things for myself" through a structured sequence of problems helped them solidify their knowledge. Many reported that they understood this mathematics better than other math they had studied, and felt they would be better able to recall—or reconstruct—it later. Interactions with other students in small groups made them aware that there are many ways to approach and solve a problem, revealed the advantages of collaboration, and made the learning process more enjoyable. Spontaneous conversations about mathematics with the instructor provided a important lessons about intellectual risk-taking and a valuable model of mathematical thinking. We feel that these results are strongly encouraging, especially considering that the guided discovery method is still on a pedagogical shake-down cruise, with neither instructor nor students fully adept at the process, while the lecture method—familiar to all—has been refined over centuries.

Feedback from students suggests that attention to a few matters of classroom management may improve the efficacy of the method, perhaps even converting the 20% of students in these two courses who were not satisfied customers. First, the pace of the course needs to be flexible enough to challenge the more advanced students without terrifying the less well prepared. Second, the instructor needs to be active throughout the term in maintaining groups that are congenial, diverse, and closely matched in competence. Finally, self-discovery needs to be accompanied by a sharing of received wisdom. No student questioned the value of figuring things out for themselves—it builds confidence and nurtures intuitions. But for students, the path they are discovering is, as one put it, through the darkness. They lack the overview to see where they are headed or to perceive connections among what they have learned. And sometimes, when the material is hard, they may need a lecture-style perspective to provide another purchase on the concepts. The well placed mini-lecture will help them organize their knowledge more effectively and reassure them that their guide will help them over the rough spots if they need it.