

posito $k = 0,05445$. Hinc autem erit $\frac{1}{2}\delta - \gamma = -0,02302$
 Cum autem iam ante inuentum esset $\frac{1}{2}\delta - \gamma = -0,01742$
 erit reuera $\frac{1}{2}\delta - \gamma = -0,00560$. Tum vero inuenimus:
 $\gamma = 1,40673$, vnde erit $\frac{1}{2}\delta = 1,40113$, et $\delta = 2,80226$
 hincque $2\gamma - \frac{1}{2}\delta = -1,38993$ et $\frac{2\gamma - \frac{1}{2}\delta}{m} = -0,00794$.
 Verum ex cognita ratione motus medi ad motum ano-
 maliae est $Q = 1,0085272$, vnde $\kappa = 1,0089562$
 Verum esse debet $\kappa = 1 + \frac{3 + 4\mu + \delta}{4m}$; vnde foret
 $0,0089562 = 0,008289 + \frac{\mu}{m}$; ideoque $\frac{\mu}{m} = 0,000667$
 qui valor cum sit tam exiguus, merito dubitamus,
 num μ non prorsus sit $= 0$.

CAPUT

CAPUT VII.

CORRECTIO INAEQUALITATUM LUNAE,
 ANTE INVENTARUM.

§. 100.

Quoniam nunc quidem valores litterarum γ et δ
 iam inuenimus, vt eos pro proxime veris habere
 queamus, ex iis coefficientes terminorum, quibus
 inaequalitates lunae continentur, accuratius definire po-
 terimus. Cum enim sit $\gamma = 1,40673$ et $\delta = 2,80226$,
 colligamus hic in vnum omnes formulas, quas hactenus
 pro inueniendis coefficientibus assumis eliciimus. Po-
 fueramus autem:

$$\begin{aligned}
 & \mathcal{R}d^2 = \mathcal{A} \text{ cof } 2\gamma + a k k \text{ cof } 2\gamma + \mathcal{B} \text{ cof } 4\gamma + b k k \text{ cof } 4\gamma \\
 & + \mathcal{C} k \text{ cof } \gamma + \mathcal{D} k \text{ cof } (2\gamma - \gamma) + \mathcal{E} k \text{ cof } (4\gamma - \gamma) \\
 & + \mathcal{F} k \text{ cof } (2\gamma + \gamma) + \mathcal{G} k \text{ cof } (4\gamma + \gamma) \\
 & + \mathcal{H} k k \text{ cof } 2\gamma + \mathcal{I} k^2 \text{ cof } (2\gamma - 2\gamma) + \mathcal{J} k^2 \text{ cof } (4\gamma - 2\gamma) \\
 & + \mathcal{K} k^2 \text{ cof } (2\gamma + 2\gamma) + \mathcal{L} k^2 \text{ cof } (4\gamma + 2\gamma) \\
 & v = A \text{ cof } 2\gamma + a k k \text{ cof } 2\gamma + B \text{ cof } 4\gamma + b k k \text{ cof } 4\gamma \\
 & + D k \text{ cof } (2\gamma - \gamma) + F k \text{ cof } (4\gamma - \gamma) \\
 & + E k \text{ cof } (2\gamma + \gamma) + G k \text{ cof } (4\gamma + \gamma) \\
 & + H k k \text{ cof } 2\gamma + J k k \text{ cof } (2\gamma - 2\gamma) + L k k \text{ cof } (4\gamma - 2\gamma) \\
 & + K k k \text{ cof } (2\gamma + 2\gamma) + M k k \text{ cof } (4\gamma + 2\gamma)
 \end{aligned}$$

§. 101.

§. 101. Hinc posito $x = \sqrt{\left(x + \frac{3 + 4k + \delta}{2m}\right)}$ collegimus fore

$$\begin{aligned} \frac{d\phi}{dx} = & x + \frac{A(3kA+2\mathfrak{N})}{2x^4} + \frac{D(3kD+2\mathfrak{D})}{2x^4} k k + \frac{A(3ka+a)}{2x^4} k k \\ & + \frac{a(3kA+2\mathfrak{N})}{2x^4} k k - \frac{(2kA+\mathfrak{N})}{m} \operatorname{cof} 2\eta - \frac{(2kB+\mathfrak{S})}{m} \operatorname{cof} 4\eta \\ & + \frac{A(3kA+2\mathfrak{N})}{2x^4} \operatorname{cof} 4\eta - \frac{\mathfrak{E}}{m} k \operatorname{cof} r - \frac{(2ka+a)}{m} k^2 \operatorname{cof} 2\eta \\ & - \frac{(2kb+\delta)}{m} k^2 \operatorname{cof} 4\eta + \frac{D(3kA+2\mathfrak{N})+A(3kD+2\mathfrak{D})}{2x^4} k \operatorname{cof} r \\ & - \frac{(2kD+\mathfrak{D})}{m} k \operatorname{cof} (2\eta-r) - \frac{(2kF+\mathfrak{S})}{m} k \operatorname{cof} (4\eta-r) \\ & + \frac{D(3kA+2\mathfrak{N})+A(3kD+2\mathfrak{D})}{2x^4} k \operatorname{cof} (4\eta-r) \\ & - \frac{(2kE+\mathfrak{E})}{m} k \operatorname{cof} (2\eta+r) - \frac{(2kG+\mathfrak{S})}{m} k \operatorname{cof} (4\eta+r) \\ & - \frac{(2kI+\mathfrak{S})}{m} k k \operatorname{cof} (2\eta-2r) - \frac{(2kH+\mathfrak{D})}{m} k k \operatorname{cof} 2r \\ & + \frac{J(3kA+2\mathfrak{N})+A(3kI+2\mathfrak{S})}{2x^4} k^2 \operatorname{cof} 2r \\ & - \frac{(2kK+\mathfrak{S})}{m} k k \operatorname{cof} (2\eta+2r) - \frac{(2kL+\mathfrak{D})}{m} k^2 \operatorname{cof} (4\eta-2r) \\ & - \frac{D(3kD+2\mathfrak{D})+J(3kA+2\mathfrak{N})+A(3kI+2\mathfrak{S})}{2x^4} k^2 \operatorname{cof} (4\eta-2r) \\ & - \frac{(2kM+\mathfrak{M})}{m} k^2 \operatorname{cof} (4\eta+2r) \end{aligned}$$

§. 102.

§. 102. Si iam ponamus

$$\begin{aligned} x + \frac{A(3kA+2\mathfrak{N})}{2x^4} + \frac{D(3kD+2\mathfrak{D})}{2x^4} k k \\ + \frac{A(3ka+a)}{2x^4} k k + \frac{a(3kA+2\mathfrak{N})}{2x^4} k k \\ - \frac{1}{x} = a; \text{ ut sit neglectis terminis admodum exiguis} \\ \frac{d\eta}{dx} = a - \frac{(2kA+\mathfrak{N})}{m} \operatorname{cof} 2\eta - \frac{(2n+\mathfrak{E})}{m} k \operatorname{cof} r \\ - \frac{(2kD+\mathfrak{D})}{m} k \operatorname{cof} (2\eta-r) - \frac{(2kE+\mathfrak{E})}{m} k^2 \operatorname{cof} (2\eta+r) \end{aligned}$$

ex superioribus capitibus repetimus has determinaciones:

$$\begin{aligned} 2a\mathfrak{N} = -\frac{3}{2}A + \frac{\mathfrak{N}(3kA+\mathfrak{N})}{m} \\ 4a\mathfrak{S} = -\frac{3A}{2m} + \frac{\mathfrak{N}(3kA+\mathfrak{N})}{m} \\ \mathfrak{E} = \frac{\mathfrak{N}(2kD+\mathfrak{D})}{m} - \frac{\mathfrak{D}-\mathfrak{E}(2kA+\mathfrak{N})}{m} - \frac{\mathfrak{N}(2kE+\mathfrak{E})}{m} - \frac{3(D-E)}{2m} \\ (2a-1)\mathfrak{D} = -3 + \frac{(2n+\mathfrak{E})}{m} \mathfrak{N} \\ (2a+1)\mathfrak{E} = -3 + \frac{(2n+\mathfrak{E})}{m} \mathfrak{N} \\ (4a-1)\mathfrak{S} = \frac{2(2k+\mathfrak{E})\mathfrak{S}}{m} + \frac{\mathfrak{N}(2kD+\mathfrak{D})}{m} + \frac{\mathfrak{D}(2kA+\mathfrak{N})}{m} - \frac{3(2A+D)}{2m} \\ (4a+1)\mathfrak{E} = \frac{2(2n+\mathfrak{E})\mathfrak{S}}{m} + \frac{\mathfrak{N}(2kE+\mathfrak{E})}{m} + \frac{\mathfrak{E}(2kA+\mathfrak{N})}{m} - \frac{3(2A+E)}{2m} \\ 2a\mathfrak{N} = -\frac{3}{2} + \frac{\mathfrak{D}+\mathfrak{E}(2n+\mathfrak{E})}{m} + \frac{2b(2kA+\mathfrak{N})}{m} \\ + \frac{2\mathfrak{S}(2kD+\mathfrak{D})}{m} + \frac{\mathfrak{D}(2kE+\mathfrak{E})}{m} \end{aligned}$$

4ab

$$4ab = -\frac{3D}{m} + \frac{2(\mathcal{F}+\mathcal{G})(2n+\mathcal{E})}{m} + \frac{\mathcal{E}(2kD+\mathcal{D})}{m}$$

$$+ \frac{a(2kA+\mathcal{H})}{m} + \frac{\mathcal{D}(2kE+\mathcal{E})}{m}$$

$$2\mathcal{D} = \frac{3(2D+\mathcal{J})}{2m} + \frac{\mathcal{E}(2kD+\mathcal{D})}{m} + \frac{\mathcal{H}(2k\mathcal{J}+\mathcal{G})}{m} - \frac{\mathcal{G}-\mathcal{H}(2kA+\mathcal{H})}{m}$$

$$2(a-1)\mathcal{G} = -\frac{1}{2} - \frac{3(H-L)}{2m} + \frac{3\mathcal{H}}{2m} + \frac{\mathcal{H}(2kH+\mathcal{D})}{m}$$

$$+ \frac{\mathcal{D}(2n+\mathcal{E})}{m} + \frac{2\mathcal{E}(2kA+\mathcal{H})}{m}$$

$$2(a+1)\mathcal{H} = -\frac{1}{2} - \frac{3H}{2m} + \frac{3\mathcal{H}}{2m} + \frac{\mathcal{H}(2kH+\mathcal{D})}{m}$$

$$+ \frac{\mathcal{E}(2n+\mathcal{E})}{m} + \frac{2\mathcal{E}(2kD+\mathcal{D})}{m}$$

$$2(2a-1)\mathcal{E} = -\frac{3(2D+\mathcal{J})}{2m} + \frac{3\mathcal{H}}{2m} + \frac{2\mathcal{E}(2kH+\mathcal{D})}{m} + \frac{2\mathcal{E}(2n+\mathcal{E})}{m}$$

$$+ \frac{\mathcal{H}(2k\mathcal{J}+\mathcal{G})}{m} + \frac{\mathcal{D}(2kD+\mathcal{D})}{m} + \frac{\mathcal{G}(2kA+\mathcal{H})}{m}$$

$$2(2a+1)\mathcal{H} = \dots + \frac{3\mathcal{H}}{2m} + \frac{2\mathcal{E}(2kH+\mathcal{D})}{m} + \frac{\mathcal{H}(2kA+\mathcal{H})}{m}$$

§. 103. Antequam vltimis progrediamur, sequentes notanda sunt novae denominationes

$$E' =$$

$$\frac{(2kD+\mathcal{D})}{m}$$

$$\frac{2kE+\mathcal{E}}{m}$$

$$\frac{\mathcal{H}(2kA+\mathcal{H})}{m}$$

$$\frac{2kH+\mathcal{D}}{m}$$

$$\frac{kA+\mathcal{H}}{m}$$

$$\frac{kH+\mathcal{D}}{m}$$

$$\frac{2\mathcal{E}(2n+\mathcal{E})}{m}$$

$$\frac{k(2kA+\mathcal{H})}{m}$$

$$\frac{\mathcal{H}(2kA+\mathcal{H})}{m}$$

$$+ \mathcal{H}$$

sequentes,

$$E' =$$

$$F' = (2a+1)E - \frac{(2n+\mathcal{E})}{m}A$$

$$F' = (4a-1)F - \frac{4B}{m} \frac{A(2kD+\mathcal{D})-D(2kA+\mathcal{H})}{m}$$

$$G' = (4a+1)G - \frac{4B}{m} \frac{A(2kE+\mathcal{E})-E(2kA+\mathcal{H})}{m}$$

$$d' = 2aa - \frac{2b(2kA+\mathcal{H})}{m} - \frac{(D+E)(2n+\mathcal{E})}{m}$$

$$- \frac{2F(2kD+\mathcal{D})}{m} + \frac{D(2kF+\mathcal{G})}{m}$$

$$d' = 4ab - \frac{a(2kA+\mathcal{H})}{m} - \frac{A(2ka+a)}{m} - \frac{D(2kE+\mathcal{E})}{m}$$

$$- \frac{E(2kD+\mathcal{D})}{m} - \frac{2(F+G)(2n+\mathcal{E})}{m}$$

$$H' = 2H + \frac{D\mathcal{E}-\mathcal{D}E-A(\mathcal{G}-\mathcal{H})+\mathcal{H}(J-K)}{m}$$

$$J' = 2(a-1)J + \frac{3A}{2m} - \frac{A(2kH+\mathcal{D})}{m} - \frac{D(2n+\mathcal{E})}{m}$$

$$- \frac{2\mathcal{E}(2kA+\mathcal{H})}{m} + \frac{A(2kL+\mathcal{E})}{m}$$

$$K' = 2(a+1)K - \frac{3A}{m} - \frac{A(2kH+\mathcal{D})}{m} - \frac{E(2n+\mathcal{E})}{m} - \frac{2G(2kD+\mathcal{D})}{m}$$

$$L' = (2a-1)L - \frac{3B}{m} - \frac{2B(2kH+\mathcal{D})}{m} - \frac{2F(2n+\mathcal{E})}{m}$$

$$- \frac{D(2kD+\mathcal{D})}{m} - \frac{A(2k\mathcal{J}+\mathcal{G})}{m} - \frac{J(2kA+\mathcal{H})}{m}$$

$$M' = 2(2a+1)M - \frac{3B}{m} - \frac{2B(2kH+\mathcal{D})}{m} - \frac{2G(2n+\mathcal{E})}{m} - \frac{E(2kE+\mathcal{E})}{m}$$

§. 104. Nunc vt terminos completos obtineamus, falem eos qui angulos 24 et r involunt, notandam

$$M' =$$

est in nostris aequationibus sin 2γ et cof 2γ non per 3/2 (1+2kk) sed per 3/2 (1+2kk+3/2 ee) esse multiplicatos. Hinc cum sit fere 3/2 ee = 3/2 kk, loco kk hic scribi oportebit 3/2 kk, vnde in valore ipsius α pro 3 scripsi 3·3/2 seu 3 3/2. Deinde vt in his terminis quoque rationem habemus inclinationis orbitae, cuius medius valor sit = ε, ponamus 3/2 (nn + 2 + 3μ + γ) tang ε² = f = 3/2 (3/2 nn nn - 3/2 nn - 3/2 + γ - 3/2 δ) tang ε² ob μ = 3/2 (nn-1) nn - 3/2 - 3/2 δ, erique nostra aequatio:

$$\begin{aligned} \frac{d\delta v}{d\gamma} &= \frac{1}{2} \delta - \gamma + \frac{1}{2} kh - \gamma k \operatorname{cof} r + \frac{3}{2} kh \operatorname{cof} 2\gamma + \frac{1}{2} \operatorname{cof} 2\gamma + \frac{1}{2} kh \operatorname{cof} 2\gamma \\ &+ f + \frac{1}{2} f kh + f k \operatorname{cof} r + \frac{1}{2} f kh \operatorname{cof} 2\gamma \\ &+ 3k \operatorname{cf} (2\gamma - r) + 3k \operatorname{cf} (2\gamma + r) + \frac{1}{2} k k \operatorname{cof} (2\gamma - 2r) + \frac{1}{2} k k \operatorname{cof} (2\gamma + 2r) \\ &- 2\gamma \sqrt{Rd} - v \left(1 - \frac{1}{2} kk + \frac{2f}{nn} + \frac{fkh}{nn} - \frac{(2\gamma - \frac{1}{2} \delta)}{nn} \right) \\ &+ v \left(3\mu n + \frac{3}{2nn} - \frac{2f}{nn} + \frac{(2\gamma - \frac{1}{2} \delta)}{nn} \right) k \operatorname{cof} r \\ &+ v \left(\frac{1}{2} - \frac{f}{nn} \right) k^2 \operatorname{cof} 2r \\ &+ v \left(\frac{3}{2nn} \operatorname{cof} 2\gamma + \frac{3k}{nn} \operatorname{cof} (2\gamma - r) + \frac{3k}{nn} \operatorname{cof} (2\gamma + r) \right) \\ &+ \frac{1}{nn} (\sqrt{Rd})^2 + \frac{6v}{nn} \sqrt{Rd} + \frac{3vv}{nn} - \frac{3vv}{nn} k \operatorname{cof} r \end{aligned}$$

§. 105. Sic breuitatis gratia:

$$\begin{aligned} 1 + \frac{2f}{nn} \frac{(2\gamma - \frac{1}{2} \delta)}{nn} &= g; \quad 3\mu n + \frac{3}{2nn} - \frac{2f}{nn} + \frac{(2\gamma - \frac{1}{2} \delta)}{nn} = h \\ \text{et } 1 - \frac{1}{2} kk + \frac{2f}{nn} + \frac{fkh}{nn} - \frac{(2\gamma - \frac{1}{2} \delta)}{nn} &= c, \text{ quo} \\ &\text{termino} \end{aligned}$$

termino in angulis ex 2 γ et r compositis veniunt: erique

$$\begin{aligned} \frac{d\delta v}{d\gamma} &= \frac{1}{2} \delta - \gamma + \frac{1}{2} kh + f + \frac{1}{2} f kh + \frac{3A}{4nn} + \frac{3akk}{4nn} \\ &+ \frac{3Dkk}{2nn} + \frac{3Ekk}{2nn} + \frac{3Gkk}{2nn} + \frac{3Qkk}{2nn} + \frac{3Rkk}{2nn} + \frac{3ADkk}{2nn} \\ &+ \frac{3AA}{nn} + \frac{3DDkk}{nn} + \frac{3AA}{2nn} + \frac{3Ekk}{2nn} \\ &+ \operatorname{cof} 2\gamma \left[\frac{1}{2} - 2\mu \mathfrak{N} - 6A \right] \\ &+ kh \operatorname{cof} 2\gamma \left\{ \frac{1}{2} - 2\mu \mathfrak{N} - 6A + \frac{1}{2} bD + \frac{1}{2} bE + \frac{cD}{nn} + \frac{3cE}{nn} \right. \\ &+ \operatorname{cof} 4\gamma \left\{ -2\mu \mathfrak{B} - 6B + \frac{3A}{4nn} + \frac{\mathfrak{N}\mathfrak{N}}{2nn} + \frac{3A\mathfrak{N}}{nn} + \frac{3AA}{2nn} \right. \\ &+ kh \operatorname{cof} 4\gamma \left\{ -2\mu b - 6b + \frac{1}{2} bF + \frac{1}{2} bG + \frac{3E}{2nn} + \frac{3D}{2nn} \right. \\ &\quad \left. \left. + \frac{\mathfrak{N}^2}{nn} + \frac{\mathfrak{D}\mathfrak{E}}{nn} + \frac{3DE}{2nn} + \frac{3Aa}{nn} + \frac{3Aa}{2nn} + \frac{3Qa}{2nn} \right. \right. \\ &+ k \operatorname{cof} r \left\{ -\gamma + f - 2x\mathfrak{C} + \frac{3D}{4nn} + \frac{3E}{4nn} + \frac{3A}{nn} + \frac{\mathfrak{N}\mathfrak{D}}{nn} + \frac{\mathfrak{N}\mathfrak{E}}{nn} \right. \\ &\quad \left. \left. + \frac{3A\mathfrak{D}}{nn} + \frac{3A\mathfrak{E}}{nn} + \frac{3\mathfrak{N}D}{nn} + \frac{3\mathfrak{N}E}{nn} + \frac{3AD}{nn} + \frac{3AE}{nn} - \frac{3AA}{2nn} \right. \right. \\ &+ k \operatorname{cof} (2\gamma - r) \left\{ 3 - 2x : \mathfrak{D} - 6D + \frac{1}{2} bA + \frac{3F}{4nn} + \frac{\mathfrak{N}\mathfrak{E}}{nn} + \frac{3A\mathfrak{C}}{nn} \right. \\ &+ k \operatorname{cof} (2\gamma + r) \left\{ 3 - 2x \mathfrak{E} - 6E + \frac{1}{2} bA + \frac{3G}{4nn} + \frac{\mathfrak{N}\mathfrak{E}}{nn} + \frac{3A\mathfrak{C}}{nn} \right. \\ &\quad \left. \left. + \frac{3A\mathfrak{E}}{nn} + \frac{3A\mathfrak{D}}{nn} + \frac{3AE}{2nn} + \frac{3AD}{2nn} \right. \right. \end{aligned}$$

M 3

VI.
$$-y + f - 2xG + \frac{3(D+E)}{4mn} + \frac{3A}{2mn} - \frac{3AA}{2mn} + \frac{3(D+E)}{mn}$$

$$+ \frac{3A(D+E)}{mn} + \frac{3(D+E)}{mn} + \frac{3(D+E)}{mn}$$

$$- C + \frac{A(2xD+D)}{mn} + \frac{A(2xE+E)}{mn} + \frac{(D+E)(2xA+3A)}{mn}$$

VII.
$$3 - 2xG - 6D + \frac{1}{2}bA + \frac{3F}{4mn} + \frac{6(3+3A)}{mn}$$

$$= - (2x-1)D' + \frac{A(2x+3A)}{mn}$$

VIII.
$$3 - 2xG - 6E + \frac{1}{2}bA + \frac{3G}{4mn} + \frac{6(3+3A)}{mn}$$

$$= - (2x+1)E' + \frac{A(2x+3A)}{mn}$$

IX.
$$- 3xG - 6F + \frac{1}{2}bB + \frac{3A}{2mn} + \frac{3D}{4mn} + \frac{3D}{mn} + \frac{3AD}{mn}$$

$$+ \frac{3AD}{mn} + \frac{3AD}{mn} - \frac{3AA}{4mn} = - (4x-1)F + \frac{4B'}{mn}$$

$$+ \frac{A(2xD+D)}{mn} + \frac{D'(2xA+3A)}{mn}$$

X.
$$- 2xG - 6G + \frac{1}{2}bB + \frac{3A}{2mn} + \frac{3E}{4mn} + \frac{3E}{mn}$$

$$+ \frac{3AE}{mn} + \frac{3AE}{mn} + \frac{3AE}{mn} - \frac{3AA}{4mn} = - (4x+1)G + \frac{4B'}{mn}$$

$$+ \frac{A(2xE+E)}{mn} + \frac{E'(2xA+3A)}{mn}$$

XI.

XI.
$$\frac{1}{2} + \frac{1}{2}f - 2xG - 6H + \frac{3(D+E)}{2mn} + \frac{3(J+K)}{4mn}$$

$$- \frac{3AD}{2mn} + \frac{6E}{2mn} - \frac{3AE}{2mn} + \frac{DE}{mn} + \frac{3DE}{mn} + \frac{3DE}{mn}$$

$$+ \frac{3G}{mn} + \frac{3GJ}{mn} + \frac{3AG}{mn} + \frac{3AJ}{mn} = - 2H' + \frac{A(2xJ+3G)}{mn}$$

$$+ \frac{E(2xD+D)}{mn} + \frac{D'(2xE+E)}{mn} + \frac{(J+K)(2xA+3A)}{mn}$$

XII.
$$\frac{1}{2} - 2xG - 6J + \frac{1}{2}bD + \left(\frac{1}{2} - \frac{f}{2mn}\right)A + \frac{3H}{4mn}$$

$$+ \frac{6(3+3A)}{mn} + \frac{3GD}{mn} = - 2(x-1)J' + \frac{3A'}{2mn}$$

$$+ \frac{D'(2x+3A)}{mn} + \frac{A(2xH+3)}{mn} + \frac{2I'(2xA+3A)}{mn}$$

XIII.
$$\frac{1}{2} - 2xG - 6K + \frac{1}{2}bE + \left(\frac{1}{2} - \frac{f}{2mn}\right)A + \frac{3H}{4mn}$$

$$+ \frac{6(3+3A)}{mn} + \frac{3GE}{mn} = - 2(x+1)K' + \frac{3A'}{2mn}$$

$$+ \frac{E'(2x+3A)}{mn} + \frac{A(2xH+3)}{mn} + \frac{2G'(2xD+D)}{mn}$$

XIV.
$$- 2xG - 6L + \frac{1}{2}bF + \left(\frac{1}{2} - \frac{f}{2mn}\right)B - \frac{3AD}{2mn}$$

$$+ \frac{3D}{2mn} + \frac{3J}{4mn} + \frac{3D}{mn} + \frac{3DD}{2mn} = - 2(2x-1)L' + \frac{3B'}{mn}$$

$$+ \frac{2F'(2x+3A)}{mn} + \frac{A(2xJ+3)}{mn} + \frac{J'(2xA+3A)}{mn} + \frac{D'(2xD+D)}{mn}$$

XV.

$$\begin{aligned}
& -2x\mathfrak{M} - \epsilon M + \frac{1}{2}bG + \left(\frac{1}{2} - \frac{f}{2m}\right)B - \frac{3AE}{2mn} \\
& + \frac{3E}{2mn} + \frac{3K}{4mx} + \frac{\mathcal{C}\mathcal{C}}{2mn} + \frac{3\mathcal{C}E}{mn} + \frac{3EE}{2mn} \\
& - 2(2\alpha+1)M' + \frac{3B'}{n} + \frac{2G'(2n+\mathcal{C})}{n}
\end{aligned}$$

§. 107, Nunc antequam hos valores invenire queamus, verus valor ipsius α intelligari debet: quod fiet ex valore integrali ipsius Φ , qui si vti §. 98. ponatur

$$\begin{aligned}
\Phi &= O' + \mathfrak{M}' \sin 2\eta + \text{etc. obinebitur.} \\
& + \frac{A(3kA+2\mathfrak{M})}{2n^4} + \frac{D(3kD+2\mathcal{D})}{2n^4} + \frac{A(3kA+2\mathfrak{M})}{2n^4} + \frac{D(3kD+2\mathcal{D})}{2n^4} \\
& = O - (\mathfrak{M}' + d'/k) \frac{(2kA+\mathfrak{M})}{mn} - \mathcal{D}' \frac{(2kD+\mathcal{D})}{mn} + \frac{1}{n}
\end{aligned}$$

vbi ex observationibus constat esse $O = 1,0085272$

Proxime autem esse supra invenimus esse:

$\mathfrak{M}' = 0,01$	$\mathfrak{M} = -0,80$	$A = -1,25$
$d' = 0,05$	$d = -2,05$	$k = -12,60$
$\mathcal{D}' = -0,44$	$\mathcal{D} = -3,60$	$D = 34,25$

atque $k = 1,0085$; $k' = 0,003$; $mn = 175,71795$

unde invenimus $O + 0,000649 = \alpha + \frac{1}{n} = k + 0,000285$

§. 108. Cum nunc sit $O = 1,0085272$, erit $\alpha + \frac{1}{n} = 1,009176$, et ob $\frac{1}{n} = 0,075438$, habebitur verus valor:

$\alpha = 0,933738$ et $1/\alpha = 0,9702255$

atque $k = 1,008991$ et $1/k = 0,0038874$

Hinc

$$\begin{aligned}
& - \frac{3AE}{2mn} \\
& \frac{E}{n} \\
& \frac{B}{n}
\end{aligned}$$

quaerod fiet acur

$$\begin{aligned}
& \mathfrak{M}'k = \\
& \alpha + \frac{1}{n} = \\
& 1,5272
\end{aligned}$$

25	71795
60	00285
25	

valor: $\frac{1}{n} =$

Hinc

Hinc iam primo obinemus:

$$\mathfrak{M} = -0,80313 \quad 1-\mathfrak{M} = 9,9047898$$

Deinde cum sit satis prope $\mathcal{C} = -0,07465$, erit

$$\frac{(2n+\mathcal{C})}{mn} = 0,147037 \text{ et } \frac{2n+\mathcal{C}}{mn} = 9,1674260$$

$$\begin{aligned}
\mathcal{D} &= -3,593620 \dots 1-\mathcal{D} = 0,5555310 \\
\mathcal{C} &= -1,087320 \dots 1-\mathcal{C} = 0,0363580
\end{aligned}$$

atque porro ex valore ipsius A proxime cognitio erit

$$\mathfrak{B} = +0,006967 \dots 1-\mathfrak{B} = 7,8430540$$

et quia est satis prope $B = 0,0128$, erit $A = 2kA - 0,000720$

$$\frac{1}{2} + 1,62172 - \epsilon A = -4kA + 0,00144\alpha + 0,000374\epsilon A$$

$$\begin{aligned}
& + 0,01247 - \epsilon B = -16kAB + 0,00570\alpha - 0,038032kA \\
& + 0,000014
\end{aligned}$$

§. 109. Nunc primum quaeri debent valores litterarum f, b et l : et cum sit $\alpha = 5'$, g' et $2\gamma - \frac{1}{2}b = -1,3899$ proxime, reperientur

$$\begin{aligned}
f &= 1,093757 \quad \text{et} \quad 1/f = 0,9389208 \\
b &= 3,0423 \quad \dots \quad l/b = 0,4832020 \\
l &= 1,01591 \quad \dots \quad l/b = 0,0068560
\end{aligned}$$

hincque erit

$$2,4720 A = -3,11947 - 0,14\alpha \quad B = 12,9369 \quad B = +0,07684 - 0,0355 A$$

unde concluditur fore:

$$\begin{aligned}
A &= -1,262463 \quad \dots \quad 1-A = 0,1012186 \\
B &= +0,009404 \quad \dots \quad 1/B = 7,9733114
\end{aligned}$$

Porto

Porro vero est

$$D' = 0,867676 D + 0,185628$$

$$E' = 2,867676 E + 0,185628$$

et $A' = -2,35859$. . . $L-A' = 0,3726530$

§. 110. Ex his valoribus aequationes VII et VIII induent has formas,

$$3 + 725185 - 6D = 1,92040 - 0,00244 + 0,01704 =$$

$$-0,75286 D = 0,16106 - 0,34680$$

$$3 + 2,19420 - 6E = 1,92040 + 0,00006 + 0,01704 =$$

$$-8,22357 E = 0,53232 - 0,34680$$

unde prohibet

$$D = + 33,6600 \dots \quad / D = 1,5271130$$

$$E = - 0,5785 \dots \quad / E = 9,7623410$$

ergo $D' = 29,39153$

$$E' = -1,47347$$

Ex his transierunt sequentes formulas pro calculo fe-
quendi

$$\frac{2kA + 9}{m} = -0,01884 \quad / \frac{(2kA + 9)}{m} = 8,275051$$

$$\frac{2kB + 8}{m} = +0,000148 \quad / \frac{2kB + 8}{m} = 6,170262$$

$$\frac{2kD + 0}{m} = +0,36611 \quad / \frac{2kD + 0}{m} = 9,563604$$

$$\frac{2kE + 0}{m} = -0,01293 \quad / \frac{(2kE + 0)}{m} = 8,108292$$

§. III.

§. VII. Ex his iam porro inuenitur

$$C = -0,64383 \dots \quad / -C = 9,80877R$$

$$\text{arque} \quad C' = -0,13847$$

Porro valores litterarum S et S' determinabuntur per
has aequationes

$$(4w-1) S = 0,002049 - 0,294032 + 0,067699$$

$$+ 0,021554 - 0,287335$$

$$(4w+1) S' = 0,002049 + 0,010306 + 0,020484$$

$$+ 0,021554 + 0,004939$$

ex quibus reperitur

$$S = -0,17957 \dots \quad / -S = 9,25424R$$

$$S' = +0,01253 \dots \quad / S' = 8,097936$$

arque

$$F' = (4w-1) F - 0,00283 + 0,46219 + 0,63411 =$$

$$(4w-1) F + 1,09347$$

$$G' = (4w+1) G - 0,00283 - 0,01620 - 0,01090 =$$

$$(4w+1) G - 0,02993$$

unde aequationes IX et X prohibent.

$$+ 0,36238 - 1,01591 F + 0,00353 - 0,00680 - 1,00349 =$$

$$- (4w-1)^2 F - 2,98415 - 0,86349 + 0,00337 - 0,55370$$

$$- 0,02528 - 1,01591 G + 0,00353 - 0,00680 + 0,04634 =$$

$$- (4w+1)^2 G + 0,14473 + 0,03027 + 0,00337 + 0,02776$$

$$\text{feu} \quad 6,43486 F = - 3,75359$$

$$21,40769 G = + 0,18534$$

§. 110. Hinc prodeunt sequentes valores correcti

pro F et G,

$$F = -0,58360 \dots \quad / -F = 9,76611R$$

$$G = +0,00866 \dots \quad / G = 7,937400$$

N 3

Ex

III.

VIII

704 =

704 =

110 fe-

Ex formula autem sexta hinc leui calculo colligitur fore:

$$\gamma - f = 1,58161 \text{ et } \gamma = 2,67537$$

Valores autem ex F et G derivati erunt

$$F' = -0,49919 \text{ et } G' = 0,01107$$

$$\frac{2\%F + G}{m} = -0,00772 \quad \frac{L(2\%F + G)}{m} = 7,887828$$

$$\frac{2\%G + S}{m} = 0,00017 \quad \frac{L(2\%G + S)}{m} = 6,232305$$

§. 113. Nunc procedamus ad valores litterarum a et b qui erunt

$$2,867676 a = -3,75000 - 0,68827 - 0,13148 - 0,03764 b$$

$$+ 0,02776$$

$$3,735352 b = -0,57467 - 0,04913 - 0,39807 - 0,01884 a$$

$$+ 0,04611$$

unde reperitur:

$$a = -2,42686 \quad \dots \quad L-a = 0,385044$$

$$b = -0,24899 \quad \dots \quad L-b = 9,396182$$

huncque porro

$$d' = 2aa + 0,03768 b - 4,86420 + 0,16048$$

$$b' = 4ab + 0,01884 a + 0,16887 + 0,64373$$

$$+ 0,01450 a - 0,01744$$

$$\text{feu } d' = 2aa + 0,03768 b - 4,70372$$

$$b' = 4ab + 0,03334 a + 0,79516$$

§. 114. Aequationes IV et V hinc induent sequentes formas:

IV.

fore:

$$\text{IV. } + 3,75000 - 1,01591 a + 50,32200 - 0,36363 = -4aa$$

$$+ 4,89730)$$

$$- 0,07034 b + 8,78034 + 4,10500 - 0,36551 - 0,00185 a$$

$$- 0,14067 b \quad \quad \quad - 0,02996$$

$$\text{V. } + 0,50246 - 1,01591 b - 0,87350 + 0,28239 - 0,95732$$

$$- 0,02155 a$$

$$= -16aa b - 0,12447 a - 2,96860 - 0,00142 b$$

$$- 0,03517 a + 0,17723$$

$$- 0,14354$$

$$- 0,91659$$

Hinc fit

$$2,47355 a = -0,21101 b - 46,11580$$

$$12,93836 b = -0,13809 a - 2,80553$$

$$\text{et } a = -18,64200 \quad \dots \quad L-a = 1,270493$$

$$b = -0,01794 \quad \dots \quad L-b = 8,253822$$

ex quibus oriuntur:

$$d' = -39,52164 \quad \dots \quad b' = +0,10663$$

$$\text{et } \frac{2\%a + a}{m} = -0,22790 \quad \frac{L(2\%a + a)}{m} = 9,357744$$

valor autem ipsius $\frac{2\%b + b}{m}$ nullius plane erit momenti,

unde eum praetermittimus.

§. 115. Ex prima autem aequatione §. 106, colligitur

$$\frac{1}{2} \delta = \gamma - f + 0,02285$$

supra autem invenimus esse $\gamma - f = 1,58161$, sicque

$$\text{erit } \frac{1}{2} \delta = 1,60446 \text{ argue}$$

$$\delta = 3,20892 \quad \dots \quad l\delta = 0,506558$$

Nunc cum fit proxime: $\delta = -0,123$; $H = -1,033$

ideoque $\frac{2\%H + H}{m} = -0,0126$; ob $\delta = -1,453$ et

$L = +6,252$; habebimus

V.

rum

54 b

14 a

- 0,132324 \mathcal{G} = - 3,75000 + 0,00882 - 0,00088
 + 0,05336 - 0,01012
 - 0,52839 + 0,05474
 + 3,867676 \mathcal{R} = - 3,75000 + 0,00882 - 0,09088
 + 0,01012
 - 0,15987 + 0,00917

Hinc reperitur

\mathcal{G} = 32,05945 . . . $l\mathcal{G}$ = 1,505956
 \mathcal{R} = - 1,02714 . . . $l\mathcal{R}$ = 0,011629

§. 116. Hinc vltimus progrediendo habebimus.

$\mathcal{J}' = 2(a-1)\mathcal{J} + 0,28571 - 4,94924 + 0,23502 =$
 $- 0,01591 - 0,08015$
 $2(a-1)\mathcal{J} - 4,52457$
 $\mathcal{K} = 2(a+1)\mathcal{K} + 0,28571 + 0,08507 - 0,00317 =$
 $- 0,01591$
 $2(a+1)\mathcal{K} + 0,35170$

vnde aequationes XII et XIII funt

+ 3,75000 - 64,69550 - 1,01591 \mathcal{J} + 51,28180
 $- 0,94292 + 0,00321 - 0,36999 =$
 $- 0,00441)$
 $- 4(a-1)^2\mathcal{J} - 0,59871 - 0,26690 + 4,32164$
 $+ 0,02972 - 0,47004$
 $+ 3,75000 + 2,07275 - 1,01591\mathcal{K} - 0,88006$
 $- 0,94292 + 0,00321 + 0,00636 =$
 $- 0,00441)$
 $- 4(a+1)^2\mathcal{K} - 1,36026 - 0,26690 - 0,21665$
 $+ 0,02972 + 0,00810$
 ex quibus colligitur fore
 $\mathcal{J} = - 14,09600 . . . l\mathcal{J} = 1,14906$
 $\mathcal{K} = - 0,41676 . . . l\mathcal{K} = 9,619888$
 Hinc

Hinc $\mathcal{J}' = - 2,65933 . . . \mathcal{K}' = - 1,26020$
 atque $\frac{2\mathcal{K}\mathcal{J} + \mathcal{G}}{\mathcal{M}} = + 0,02057 . l\frac{2\mathcal{K}\mathcal{J} + \mathcal{G}}{\mathcal{M}} = 8,313172$
 $\frac{2\mathcal{K}\mathcal{K} + \mathcal{R}}{\mathcal{M}} = - 0,01063 . l\frac{2\mathcal{K}\mathcal{K} + \mathcal{R}}{\mathcal{M}} = 8,026598$

§. 117. Quæramus iam valorem ipsius \mathcal{J} , ex aequatione

$2\mathcal{J} = - 0,45434 - 0,39771 - 0,01652 + 0,62330$
 erit $\mathcal{J} = - 0,12264 . . . l\mathcal{J} = 9,088632$
 hincque reperitur: $\mathcal{H}' = 2\mathcal{H} - 0,08914$
 vnde aequatio XI præbet:
 $2,04688 + 0,24748 - 1,01591\mathcal{H} + 0,27805 - 3,06194$
 $+ 0,36275 + 0,00118 - 0,00623 + 0,02224 + 0,03552$
 $- 0,62485 - 0,33247 - 0,83753 + 0,49710 =$
 $- 4\mathcal{H} - 0,16022 - 0,04851 - 0,53944 - 0,37802$
 $+ 0,07384$
 feu $2,98409\mathcal{H} = - 2,68053$
 Ergo $\mathcal{H} = - 0,89829 . . . l\mathcal{H} = 9,953447$
 $\mathcal{H}' = - 1,71647 ; \frac{2\mathcal{H} + \mathcal{J}}{\mathcal{M}} = - 0,01102$

§. 118. Tandem superfunct literæ \mathcal{Q} et \mathcal{M}

$2(a-1)\mathcal{Q} = - 0,45434 - 0,05280 - 0,01652 - 1,31570$
 $+ 0,00158 - 0,60396$
 $- 0,00015$
 $2(a+1)\mathcal{M} = + 0,00158 + 0,00368 + 0,01935$
 $- 0,00015$
 Hinc $\mathcal{Q} = + 5,40715 . . . l\mathcal{Q} = 0,148340$
 $\mathcal{M} = + 0,00426 . . . l\mathcal{M} = 7,629896$
 O Deinde

Deinde vero habebitur :
 $L' = 2(2a-1) L - 0,00213 + 0,17162 - 12,31170 - 0,02596$
 $+ 0,00013 = -0,26575$
 $M' = 2(2a+1) M - 0,00213 - 0,00254 - 0,00742$
 $+ 0,00013$

feu $L' = 2(2a-1) L - 12,43359$
 $M' = 2(2a+1) M - 0,01196$

§. 119. Nunc demique aggregamus aequationes

XIV et XV

XIV. $-2,83960 - 1,01591 L - 0,88774 + 0,00702 + 0,36275$
 $+ 0,28733 = -0,06016 + 0,03675 + 7,60650 =$
 $-3,0144L + 25,57666 + 0,00255 - 0,14680 + 10,75272$

XV. $+0,00860 - 1,01591 M + 0,01317 + 0,00702 - 0,00623$
 $- 0,00494 = -0,00176 + 0,00336 + 0,03360 =$
 $-32,89415 M + 0,06859 + 0,00255 + 0,00325$
 ex quibus eruitur

$L = + 13,86720 \dots / L = 1,141988$
 $M = + 0,00131 \dots / M = 7,117165$
 hincque $L' = 11,3090$ et $M' = 0,00445$

$\frac{2kL + \xi}{n} = 0,15125 \dots \frac{2kL + \xi}{n} = 9,179684$
 $\frac{2kM + \eta}{n} = 0,00004 \dots \frac{2kM + \eta}{n} = 5,592770$

Ex his valoribus nouae correctiones inueniri possunt, sed differentiae prodirent tam exiguae, vt operae pretium non sit eas investigare.

§. 120.

§. 120. His igitur valoribus inuentis, denotante iam a distantiam Lunae mediam a Terra, et eius distantia currae $= x$, cum sit $x = \frac{(1-kk)^2}{1-k \cos^2 \theta}$, erit:

#	Log. coefficient:
$\# = 1 - 0,0074991 \cos^2 \theta$	7,875009
$+ 0,0000532 \cos^4 \theta$	5,725912
$+ 0,191557k \cos^2(2\theta - r)$	9,282297
$- 0,003293k \cos^2(2\theta + r)$	7,517525
$- 0,003321k \cos^2(4\theta - r)$	7,521296
$+ 0,000049k \cos^2(4\theta + r)$	5,692584
$- 0,00511kk \cos^2 r$	7,708601
$- 0,08022kk \cos^2(2\theta - 2r)$	8,904280
$- 0,00237kk \cos^2(2\theta + 2r)$	7,375072
$+ 0,07892kk \cos^2(4\theta - 2r)$	8,897172
$+ 0,00001kk \cos^2(4\theta + 2r)$	4,872349,

vbi quidem in duobus primis terminis signum eos, qui per kk erant affecti, sumus complexi, posito $k = 0,05445$. Etiam si enim hic valor non omnino esset iustus, tamen inde in his terminis minimis nullus error nasci poterit.

§. 121.

§. 121. Porro quoque hinc ex §. 116. valorem ipsius $\frac{d\theta}{dr}$ determinabimus, quatenus a sola excentricitate orbitae lunaris pender.

$\frac{d\theta}{dr} =$	log. coeff.
1,009276	0,004010
+ 0,0195144 $\text{cof } 2r$	8,290355
— 0,0000322 $\text{cof } 4r$	5,507856
— 0,001231 $\text{cof } r$	7,090258
— 0,366103 $\text{cof } (2r-r)$	9,563604
+ 0,012832 $\text{cof } (2r+r)$	8,108292
+ 0,002829 $\text{cof } (4r-r)$	7,451633
— 0,000171 $\text{cof } (4r+r)$	6,232305
+ 0,01182 $\text{cof } 2r$	8,072618
— 0,02057 $\text{cof } (2r-2r)$	8,313172
+ 0,01063 $\text{cof } (2r+2r)$	8,026598
— 0,09883 $\text{cof } (4r-2r)$	8,994889
— 0,00004 $\text{cof } (4r+2r)$	5,592770

§. 122.

1 ipsius
e orbitae

§. 122. Cum nunc sit $\frac{d\theta}{dr} = \frac{dr}{dr} = \frac{1+2e}{m} + \frac{e}{m} \text{cof } r + \frac{3}{2m} k k \text{cof } 2r$, erit

$\frac{d\theta}{dr} =$	log. coeff.
0,933838	9,970272
+ 0,0195144 $\text{cof } 2r$	8,290355
— 0,0000322 $\text{cof } 4r$	5,507856
— 0,152101 $\text{cof } r$	9,182132
— 0,366103 $\text{cof } (2r-r)$	9,563604
+ 0,012829 $\text{cof } (2r+r)$	8,108292
+ 0,002829 $\text{cof } (4r-r)$	7,451633
— 0,000171 $\text{cof } (4r+r)$	6,232305
— 0,10133 $\text{cof } 2r$	9,005738
— 0,02057 $\text{cof } (2r-2r)$	8,313172
+ 0,01063 $\text{cof } (2r+2r)$	8,026598
— 0,09883 $\text{cof } (4r-2r)$	8,994889
— 0,00004 $\text{cof } (4r+2r)$	5,592770

quae formulae ad motum Lunae horarium tam abfolutum quam a sole adhiberi poffunt, quemadmodum illa distantiam deficiens diametro apparenti et parallelaxi horizontali inueftigandae interuit.

Q 3

☉ =

§. 23. Queramus nunc valorem integram pro longitudine Lunae Φ , quatenus a sola excentricitate orbis lunaris pendet, ac ponamus.

$$\begin{aligned}
\Phi = & 0r + \mathcal{A}' \sin 2\eta + a'k \sin 2\eta + \mathcal{B}' \sin 4\eta + b'k \sin 4\eta \\
& + \mathcal{C}' k \sin r + \mathcal{D}' k \sin (2\eta - r) + \mathcal{E}' k \sin (4\eta - r) \\
& + \mathcal{F}' k \sin (2\eta + r) + \mathcal{G}' k \sin (4\eta + r) \\
& + \mathcal{H}' k \sin 2r + \mathcal{I}' k \sin (2\eta - 2r) + \mathcal{K}' k \sin (4\eta - 2r) \\
& + \mathcal{L}' k \sin (2\eta + 2r) + \mathcal{M}' k \sin (4\eta + 2r)
\end{aligned}$$

arque sequentes obtinebimus formulas:

$$\begin{aligned}
+0,0188387 &= 2a\mathcal{A}' - \frac{\mathcal{A}'(2k\mathcal{B} + \mathcal{B})}{n} - \frac{2\mathcal{B}'(2k\mathcal{A} + \mathcal{A})}{n} \\
-0,0000370 &= 4a\mathcal{B}' - \frac{\mathcal{B}'(2k\mathcal{A} + \mathcal{A})}{n} \\
-0,001231 &= \mathcal{C}' - \frac{\mathcal{A}'(2k\mathcal{D} + \mathcal{D})}{n} + \frac{\mathcal{A}'(2k\mathcal{E} + \mathcal{E})}{n} \\
&\quad - \frac{(\mathcal{D}' + \mathcal{E}')(2k\mathcal{A} + \mathcal{A})}{n} \\
-0,366103 &= (2a-1)\mathcal{D}' - \frac{\mathcal{A}'(2n + \mathcal{D})}{n} \\
+0,012832 &= (2a+1)\mathcal{E}' - \frac{\mathcal{A}'(2n + \mathcal{E})}{n} \\
+0,002829 &= (4a-1)\mathcal{F}' - \frac{4\mathcal{B}'}{n} - \frac{\mathcal{A}'(2k\mathcal{D} + \mathcal{D})}{n} - \frac{\mathcal{D}'(2k\mathcal{A} + \mathcal{A})}{n} \\
-0,000171 &= (4a+1)\mathcal{G}' - \frac{4\mathcal{B}'}{n} - \frac{\mathcal{A}'(2k\mathcal{E} + \mathcal{E})}{n} - \frac{\mathcal{E}'(2k\mathcal{A} + \mathcal{A})}{n}
\end{aligned}$$

in pro
fite or-

$$\begin{aligned}
& k \sin 4\eta \\
& \eta - r \\
& \eta + r \\
& 1 - 2r \\
& + 2r
\end{aligned}$$

$$\begin{aligned}
+0,22790 &= 2a\mathcal{A}' - \frac{(\mathcal{D}' + \mathcal{E}')(2n + \mathcal{D})}{n} - \frac{2\mathcal{F}'(2k\mathcal{D} + \mathcal{D})}{n} \\
&\quad - \frac{2\mathcal{B}'(2k\mathcal{A} + \mathcal{A})}{n} \\
+0,00163 &= 4a\mathcal{B}' - \frac{a'(2k\mathcal{A} + \mathcal{A})}{n} - \frac{\mathcal{E}'(2k\mathcal{D} + \mathcal{D})}{n} \\
&\quad - \frac{2(\mathcal{F}' + \mathcal{G}')(2n + \mathcal{D})}{n} \\
+0,01182 &= 2\mathcal{H}' - \frac{\mathcal{E}'(2k\mathcal{D} + \mathcal{D})}{n} - \frac{\mathcal{A}'(2k\mathcal{J} + \mathcal{J})}{n} \\
&\quad - \frac{(\mathcal{G}' + \mathcal{H}')(2k\mathcal{A} + \mathcal{A})}{n} - \frac{\mathcal{D}'(2k\mathcal{E} + \mathcal{E})}{n} \\
-0,02057 &= 2(a-1)\mathcal{I}' - \frac{3\mathcal{B}'}{n} - \frac{\mathcal{D}'(2n + \mathcal{D})}{n} - \frac{\mathcal{A}'(2k\mathcal{H} + \mathcal{H})}{n} \\
&\quad - \frac{2\mathcal{E}'(2k\mathcal{A} + \mathcal{A})}{n} \\
+0,01063 &= 2(a+1)\mathcal{K}' - \frac{3\mathcal{B}'}{n} - \frac{\mathcal{E}'(2n + \mathcal{E})}{n} - \frac{\mathcal{A}'(2k\mathcal{H} + \mathcal{H})}{n} \\
&\quad - \frac{2\mathcal{G}'(2k\mathcal{D} + \mathcal{D})}{n} \\
-0,09883 &= 2(2a-1)\mathcal{L}' - \frac{3\mathcal{B}'}{n} + \frac{2\mathcal{F}'(2n + \mathcal{D})}{n} + \frac{\mathcal{A}'(2k\mathcal{J} + \mathcal{J})}{n} \\
&\quad - \frac{\mathcal{G}'(2k\mathcal{A} + \mathcal{A})}{n} - \frac{\mathcal{D}'(2k\mathcal{D} + \mathcal{D})}{n} \\
-0,00004 &= 2(2a+1)\mathcal{M}' - \frac{3\mathcal{B}'}{n} - \frac{2\mathcal{G}'(2n + \mathcal{E})}{n}
\end{aligned}$$

§. 124. Ex his eliduntur valores sequentes:

$\mathcal{M}' = +0,0100887$	$- \frac{1}{\mathcal{M}'} = 8,003837$	$a' = 0,09140$
$\mathcal{M}'' = -0,0000409$	$- \frac{1}{\mathcal{M}''} = 3,611723$	$a'' = 8,960934$
$\mathcal{G}' = +0,010146$	$- \frac{1}{\mathcal{G}'} = 8,006295$	$b' = 0,00089$
$\mathcal{D}' = -0,420226$	$- \frac{1}{\mathcal{D}'} = 9,633483$	$b'' = 6,949340$
$\mathcal{E}' = +0,0004992$	$- \frac{1}{\mathcal{E}'} = 7,698261$	$\mathcal{M} + a''k = 0,0103597$
$\mathcal{F}' = +0,0005286$	$- \frac{1}{\mathcal{F}'} = 7,723163$	$\mathcal{M} + b''k = -0,0000382$
$\mathcal{G}'' = -0,000086$	$- \frac{1}{\mathcal{G}''} = 5,935307$	$\mathcal{M} + a''k + b''k = 8,015347$
$\mathcal{H}' = +0,000420$	$- \frac{1}{\mathcal{H}'} = 7,623250$	$\mathcal{M} + b''k = 5,582063$
$\mathcal{I}' = +0,57328$	$- \frac{1}{\mathcal{I}'} = 9,7502427$	
$\mathcal{J}' = +0,00318$	$- \frac{1}{\mathcal{J}'} = 9,178488$	
$\mathcal{K}' = -0,15083$	$- \frac{1}{\mathcal{K}'} = 9,178488$	
$\mathcal{M}''' = -0,00002$	$- \frac{1}{\mathcal{M}'''} = 5,301030$	

§. 125. Pro longitudine ergo Lunae habemus
habemus hanc formulam

$\phi = \text{Confl.}$	$+ 1,0085272$	r	$0,003687$
	$+ 0,0103197$	$\sin 2\eta$	$8,015347$
	$- 0,0000382$	$\sin 4\eta$	$5,582063$
	$+ 0,010146k$	$\sin r$	$8,006295$
	$- 0,420226k$	$\sin(2\eta - r)$	$9,623483$
	$+ 0,0004992k$	$\sin(2\eta + r)$	$7,698261$
	$+ 0,005286k$	$\sin(4\eta - r)$	$7,723163$
	$- 0,000086k$	$\sin(4\eta + r)$	$5,935307$
	$+ 0,000420k$	$\sin 2r$	$7,623250$
	$+ 0,57328k$	$\sin(2\eta - 2r)$	$9,758367$
	$+ 0,00318k$	$\sin(2\eta + 2r)$	$7,402427$
	$- 0,15083k$	$\sin(4\eta - 2r)$	$9,178488$
	$- 0,00002k$	$\sin(4\eta + 2r)$	$5,301030$

§. 126.

§. 126. Quodsi ianti potissimum $k = 0,65445$, et hos
coefficientes ad minuta secunda cum partibus decimali-
bus reducimus, longitudo ϕ ka exprimitur ut fit:

$\phi = \text{Confl.}$	$+ 1,0085272$	r	log. coeff.
	$+ 2136''$	$8 \sin 2\eta$	$3,329722$
	$- 7, 8$	$\sin 4\eta$	$0,895488$
	$+ 113, 9$	$\sin r$	$2,056718$
	$- 4719, 6$	$\sin(2\eta - r)$	$3,573906$
	$+ 56, 1$	$\sin(2\eta + r)$	$1,748684$
	$+ 59, 4$	$\sin(4\eta - r)$	$1,773586$
	$- 1, 0$	$\sin(4\eta + r)$	$9,988720$
	$+ 2, 5$	$\sin 2r$	$0,409671$
	$+ 350, 6$	$\sin(2\eta - 2r)$	$2,544788$
	$- 1, 5$	$\sin(2\eta + 2r)$	$9,488848$
	$- 92, 2$	$\sin(4\eta - 2r)$	$1,964909$
	$- 0, 0$	$\sin(4\eta + 2r)$	$8,087451$

Haeque formulae praecipuae inaequalitates, quibus motus
Lunae perturbatur, continentur.

DE MOTU APOGEI LUNAE.

§. 127.

His inventis iam arduam illam de motu apogei Lunae quaestionem examinare, atque adeo decidere licebit. Quamquam enim in praecedentibus calculis vbiq; verum apogei motum, quem observationes ostendunt, introduxi, ita vt id ipsum, quod in controuersa est, assumisse videar; tamen quoniam in hunc ipsum finem terrae vim, qua Luna vrgetur, indefinitam sum contempserat, dum rationi distantiarum reciprocae duplicatae terminum indefinitum adiunxi, vnde littera μ in aequatione est inuenta, iudicium de eo apogei motu, quod Theoriae Neutonianae esse consentaneum, non erit difficile. Quod si enim valor litterae μ nihilo aequalis reperitur, hinc concludendum erit Theoriam Neutoniam esse praeter omnem periculis consentaneam; sin autem pro littera μ nonnullis pygmaea valor, Theoria ista insufficientis erit censenda.

§. 128. Motus autem apogei, quoniam huius rei in calculo nusquam mentio est facta, in ea conuenitur proportione, quam motus lunae medius ad motum anomaliae tenere est positus. Cum enim remotus lunae inaequalitatis, quae regulae Keplerianae aduersantur, longitudo lunae vera obtineatur, si eius anomalia vera r ad longitudinem apogei addatur; denotet φ longitudinem apogei, eritque longitudo vera $\varphi = r + r$, vnde

si Lunae motus inaequalitatem ad motum anomaliam tenent, quibus apogei motus non afficitur, omnibus per integrationem deducimur ad huiusmodi formam $\varphi = \text{Const.} + \text{Or}$, vnde propterea habetur longitudo apogei $\varphi = \text{Const.} + (\text{O}-1)r$. Hinc consequuntur sequentes proportionales:

- I. Vt 1 ad O-1, ita motus anomaliae lunae ad motum apogei.
- II. Vt O ad 1, ita motus lunae medius ad motum anomaliae.
- III. Vt O ad O-1, ita motus lunae medius ad motum apogei.

§. 129. Cum autem terminos illos omnes, qui finis angularum implicent, ideoque inaequalitates periodicas continent, quibus apogei motus non afficitur, omnibus per integrationem deducimur ad huiusmodi formam $\varphi = \text{Const.} + \text{Or}$, vnde propterea habetur longitudo apogei $\varphi = \text{Const.} + (\text{O}-1)r$. Hinc consequuntur sequentes proportionales:

- I. Vt 1 ad O-1, ita motus anomaliae lunae ad motum apogei.
- II. Vt O ad 1, ita motus lunae medius ad motum anomaliae.
- III. Vt O ad O-1, ita motus lunae medius ad motum apogei.

§. 130. Si observationes consulamus, valor litterae O reperitur = 1,0085272, quem etiam in calculo vbiq; adhibui; propterea quod propositum erat non tam in istum valorem a priori inquirere, quam ipsam potius Theoriam ita instituire, atque si opus fuerit, emendare, vt motus inde apogei experientiae consentaneus reuelaret. Vixisset autem Theoria stabilis, siue Neutoniana siue alia, quae ex determinato pro r stabilito valore oritur, facile erit valorem ipsius O a priori eruere, quem deinceps cum valore vero 1,0085272, conferre licebit. Vel inuento valore ipsius O, apogei motum

hunc intervallo mensis apogitici progreditur per spatium (O-1) 360°, intervallo autem mensis periodici per spatium (1- $\frac{1}{2}$) 360°. Secundum observationes autem apogeam promouetur
 vno mense apogitico per spatium 3°, 4', 11"
 vno mense periodico per spatium 3, 2, 38

§. 131. Ex calculo autem §. 107, expofito valor litterae O ex elementis ante assumtis ita definitur, vt fit
 $O + 0,000649 = k + 0,000285$ siue $O = k - 0,000364$
 Erft enim haec exigua particula 0,000364 iam ex valore ipsius & veritati consentaneae assumto est orta, tamen perspicuum est, leuem differentiam nullius hic momenti futuram fuisse. Verum littera k per Theoriam ita erat assumta, vt esse

$$k = \sqrt{\left(1 + \frac{3+4\mu+\delta}{2m}\right)}$$

vbi quidem valor ipsius m ex motu medio lunae ad motum solis relatus habetur, ita vt sine respectu ad motum apogei habitus $m = 175,71795$. Ergo pro Theoria Newtoniana est
 $k = \sqrt{\left(1 + \frac{3+\delta}{2m}\right)}$ et $O = \sqrt{\left(1 + \frac{3+\delta}{2m}\right)} - 0,000364$.

§. 132. Hic igitur per totam hanc investigationem ad inuentionem litterae δ reduci, cuius valor, vti ex superiori calculo manifestum est, a pluribus litteris et coefficientibus terminorum, quos ante erriare oportebat, pender, ita vt neglecta haec littera δ motus apogei nullo modo resse desinit quae hinc quidem vbi hanc theoriam

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 s autem

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 000364
 ex va-
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nae ad
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 Theo-
 00364.
 igitur
 or, vti
 litteris
 rebas,
 nullo
 mc lit-
 teram

teram in calculum induximus, quod factum est §. 44. haec res leuis momenti est visa; cum enim pro CC, quae erat constans per integrationem in calculum ingressa, valorem vero proximum inuenissemus $1 + \frac{3+4\mu}{2m}$, quoniam facile erat praevidere, reliquis adhibitis elementis ad motum lunae pertinentibus, hunc valorem aliquantulum immuari posse, pro vero valore ipsius $\frac{CC}{1-4k}$ potissimum $1 + \frac{3+4\mu+\delta}{2m}$. Deinde autem valor ipsius δ potissimum pender a valore litterae γ , qua vis sumus ad verum valorem constans $\frac{m}{m} = 1 + \frac{2+3\mu+\gamma}{m}$ obinendum, cum prorsus xime verus esset inuentus $= 1 + \frac{2+3\mu}{m}$.

§. 133. Ab his ergo litteris γ et δ , quae initio nullius fere usus esse videbantur, determinatio motus apogei potissimum pender, quae cum ex pluribus atque adeo omnibus inaequalitatibus lunae ab eccentricitate ortis determinari debeant, mirum sane non est, quod legitima motus apogei designatio, cum tantis implicata sit difficultatibus, tam dudum fuerit abscondita. Plerique enim, qui motum apogei ex sola Theoria concludere sunt annisi, ad omnes has inaequalitates non respexerunt, atque calculum perinde administraverunt, ac si hic litteras γ et δ neglexissent. Ac si non desinere, qui sibi persuaserunt, motum apogei cum Theoria Newtoniana consentire, si plerumque per errorem calculi seduci ad veritatem peruenisse sibi sunt visi. Quia etiam Ipse Newtonus

tonus Theoriae suae in motu apogei determinando parum tribuisse videtur.

§. 134. Hinc ex neglectu harum litterarum γ et δ , seu ex alia omissione eodem residente, factum est, ut Theoria Neutroni observationibus circa motum apogei huius institutis plane non satisfacere sit putata; quae opinio etiam ita inualuit, ut perspicacissimus quisque hanc Theoriam insufficientem pronunciaret. Atque sagacissimus Clairautus huic opinioni vehementissime erat addictus, aequam publice in contrarias partes discesserat. Eadem saltem ratione ob neglectum minutarum illarum particularum erat deceptus, qua et ego fateri cogor, me per complures annos constanter esse opinatum, ex Theoria Neutroni pro motu apogei Lunae non ultra semissem prodire, ita ut error ultra semissem exurgerens committeretur.

§. 135. Fons itaque huius erroris, qui nisi summa circumspectio adhibeatur, vix evitatur, in eo lateat, quod in calculo debita illa constantium determinatio, pro qua equidem hic litteras γ et δ adhibui, negligatur. Quemadmodum per hanc omissionem dimidius tantum apogei motus elicatur, ostendisse iuvabit. Sit igitur $\delta = 0$, atque littera illius O secundum Theoriam Neutronianam, qua est $\mu = 0$, valor erit $O = \sqrt{\left(1 + \frac{3}{2m}\right) - 0,000364}$; qui euolutus fit: $O = 1,0042592 - 0,000364$. Quare etiam si particula 0,000364 vix ex profundiori indagine nata praetermitteratur, tamen iste valor pro $O = 1,0042592$, si cum vero per observationes cognitio $O = 1,0085272$ compareretur, exacte fere dimidium motum apogei praebet;

nando pa-

um γ et δ ,
m est, ut
n apogei
quae opi-
que hanc
sagacissi-
rat addi-
ificesset.
n illarum
gor, me
Theoria
m prodiretur.
si summa
ret, quod
pro qua
Quemad-
ogei motus,
atque
am, qua
364; qui
e etiam si
zine nati-
0,0042592,
0085272
ei praebet;

bet; atque adeo haec tam accurata medietas non parum digna videtur.

§. 136. Jam videamus, quam prope valorem litterae δ adhibendo ad veritatem perducamur. Invenimus autem (115) $\delta = 3,20892$, unde prodit

$$\sqrt{\left(1 + \frac{3\delta^2}{2m}\right)} = 1,0087947.$$

qui valor iam maior est quam verus 1,0085272, sed recordandum est inde subtrahi debere 0,000364, sicque relinquatur $O = 1,0084307$, ex quo motus progressivus apogei pro intervallo mensis periodici $= 3^\circ 2' 9''$ et pro intervallo mensis periodici $= 3^\circ 0' 37''$, qui numeri duobus tantum minutis a vero differunt. Ad hunc defectum supplendum litterae μ tribui poterit valor convenientis ex formula $\mu = \frac{1}{2} (\mu^2 - 1) m$ $= \frac{1}{2} - \frac{1}{2} \delta$, unde reperitur $\mu = 0,03782$, qui valor tantillus est, ut nisi de motu apogei sit quaestio, semper pro nihilo haberi possit.

§. 137. Verum nullo modo affirmare possumus, velores illos pro γ et δ inventos ita esse absolutos, si nulla amplius correctione indigeant. Quin potius, si formulae supra exhibitae attentius perpendamus, tantum abest ut eas pro completis habere possimus, ut potius manifestum sit, omnes reliquas inaequalitates motus lunae perinde ac eas quas iam definitivimus, terminos quoque in eas suppediare. Qui etsi admodum erunt parvi, tamen omnino sufficere poterunt ad exiguum istud supplementum, quo adhuc a vero distamus, consiciendum. Cum

Cum enim sola fere inaequalitas ab angulo $2\eta - r$ pendens motum apogei a dimidio tempore auxisset, vt valor ipsius O ab 1,0042592 vsque ad 1,0084307 increvisset, nullum fere est dubium, quin leuis defectus huius numeri a vero valore 1,0085272 a reliquis inaequalitatibus proficiscatur.

§. 138. Hinc igitur concludere debemus, Theoriam Newtonianam cum motu apogei obseruato tam exacte conuenire, vt aberratio, si quidem vlla locum habeat, tam sit exigua, vt merito pro nihilo reputari possit: neque etiam calculi ope ob summam paruitatem eam certe definire licebit. Cum itaque hoc pacto Theoria Newtoniana a fortissima obiectione sit vindicata, gloria huius insignis inventi cum industriae tum candori excellentissimi Clairauti debetur, qui primus egregium hunc Theoriae consensum cum veritate detexit et publice est professus: cui ea re eo maiores debemus gratias, quod sine studio summo, quod in hac investigatione consumpsit, Theoria Newtoniana fortasse vix vnguam ab hac suspitione insufficientiae esset liberata. Arque nunc demum pleno lumine veritas istius Theoriae, cui vni Astronomiae Theoria vniuersa inniditur, fulgere est censenda, cum antea non medicoribus tenebris fuisset inuoluta.

CAPUT

— r pendisset, vt 2η incrementus huius inaequa-

Theoriam exactam haberi positam: pacto indicata, candore egregio rexit et eadem hac in-

PUT

CAPUT IX.

INVESTIGATIO INAEQUALITATUM LUNAE A SOLA EXCENTRICITATE ORBITAE SOLIS PENDENTIVM

§. 139.

Quoniam in hac investigatione excentricitas orbitae lunaris non in censum venit, inaequalitates quas seruemur partim ab anomalia vera solis s partim ab angulo 2η pendebunt. Cum igitur sit

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} + \frac{2}{n} k \cos r - \frac{2}{n} e \cos s - \frac{2}{n} k \cos(r-s) + \frac{2}{2n} k \cos 2s + \frac{1}{2n} e \cos 2s - \frac{2}{n} e k \cos(s+s)$$

hinc differentiale ds ad differentiale dr reducitur. Atque hoc quidem capite, quia ad excentricitatem Lunae non attendimus, erit

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} - \frac{2}{n} e \cos s + \frac{1}{2n} e e \cos 2s$$

§. 140. Incipiamus ergo a formulis $\int R dr$ et \mathcal{Q} , quas omiffis terminis ab angulo r pendentibus ponamus

$$\begin{aligned} \int R dr &= \mathcal{R} \cos 2\eta + \mathcal{S} e \cos s + \mathcal{Q} e \cos(2\eta - s) + \mathcal{R} e \cos(2\eta + s) \\ &+ \mathcal{C} e \cos 2s + \mathcal{E} e \cos(2\eta - 2s) + \mathcal{S} e e \cos(2\eta + 2s) \\ &= A \cos 2\eta + P e \cos s + Q e \cos(2\eta - s) + R e \cos(2\eta + s) \\ &+ S e e \cos 2s + T e e \cos(2\eta - 2s) + V e e \cos(2\eta + 2s) \end{aligned}$$

\mathcal{Q}

vbi

vbi quidem pro \mathcal{X} et A valores supra inventos completos accipi oportet, ita ut in his terminis akk et akk sint comprehensi; erit ergo

$$\mathcal{X} = 0,81033 \quad 1-\mathcal{X} = 9,908662$$

$$A = 1,31773 \quad 1-A = 0,119826$$

Valores autem hinc derivati erunt:

$$\frac{2\kappa A + \mathcal{X}}{n} = -0,019744; \quad \frac{1-(2\kappa A + \mathcal{X})}{n} = 8,995442$$

$$A' = 2,47576 \quad 1-A' = 0,393708$$

$$\mathcal{X}' = 0,01036 \quad 1-\mathcal{X}' = 8,019347$$

Terminos autem angulum quadruplum 4η involuentes hic ob summam parvitatem omisi, quoniam in combinatione cum angulo s plane ferent imperceptibiles.

§. 141. Hinc iam primo colligitur:

$$\frac{d\mathcal{P}}{d\eta} = \kappa \frac{(2\kappa A + \mathcal{X})}{n} \cos 2\eta - \frac{(2\kappa P + \mathcal{Y})}{n} \epsilon \cos s$$

$$- \frac{(2\kappa Q + \Omega)}{n} \epsilon \cos(2\eta - s) - \frac{(2\kappa R + \mathcal{N})}{n} \epsilon \cos(2\eta + s)$$

$$- \frac{(2\kappa S + \mathcal{O})}{n} \epsilon \epsilon \cos 2s$$

$$- \frac{(2\kappa T + \mathcal{Z})}{n} \epsilon \epsilon \cos(2\eta - 2s) - \frac{(2\kappa V + \mathcal{U})}{n} \epsilon \epsilon \cos(2\eta + 2s)$$

atque porro

$$\frac{d\eta}{d\tau} = \kappa \frac{(\epsilon \kappa A + \mathcal{Y})}{n} \cos 2\eta - \left(\frac{2\kappa P + \mathcal{Y}}{n} - \frac{2}{n} \right) \epsilon \cos s$$

$$- \frac{(2\kappa Q + \Omega)}{n} \epsilon \cos(2\eta - s) + \frac{(2\kappa R + \mathcal{N})}{n} \epsilon \cos(2\eta + s)$$

$$- \frac{(2\kappa S + \mathcal{O})}{n} \epsilon \cos 2s - \frac{1}{2n} \epsilon \epsilon \cos 2s$$

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Deinde quia est proxime $kk = 9\epsilon\epsilon$, erit

$$R = \frac{1}{2} (1 + 3\epsilon\epsilon) \sin 2\eta + \left(\frac{3Q}{2m} - \frac{3R}{2mn} \right) \epsilon \sin s$$

$$- \left(\frac{3}{2} - \frac{3P}{2m} \right) \epsilon \sin(2\eta - s) - \left(\frac{3}{2} - \frac{3P}{2m} \right) \epsilon \sin(2\eta + s)$$

$$+ \left(\frac{3T}{2m} - \frac{3V}{2m} \right) \epsilon \sin 2s$$

$$+ \left(\frac{3}{2} + \frac{3S}{2m} \right) \epsilon \epsilon \sin(2\eta - 2s) + \left(\frac{3}{2} + \frac{3S}{2m} \right) \epsilon \epsilon \sin(2\eta + 2s)$$

atque omnis terminis, quibus non est opus.

$$\frac{d'u}{d\tau^2} = \epsilon \cos s \left[-\frac{3}{2} - 2\kappa \eta - \epsilon P - \frac{9}{4m} A + \frac{3Q}{n} + \frac{3R}{n} \right.$$

$$\left. + \frac{3AQ}{m} + \frac{3AR}{n} + \frac{3\mathcal{Y}(Q+R)}{n} + \frac{3A(\Omega+R)}{n} \right]$$

$$+ \epsilon \cos(2\eta - s) \left[-\frac{3}{2} - 2\kappa \Omega - \epsilon Q - \frac{3A}{4m} + \frac{3P}{4m} \right.$$

$$\left. + \frac{3\mathcal{Y}}{m} + \frac{3AP}{m} + \frac{3\mathcal{Y}P}{m} + \frac{3A\mathcal{Y}}{m} \right]$$

$$+ \epsilon \cos(2\eta + s) \left[-\frac{3}{2} - 2\kappa \mathcal{N} - \epsilon R - \frac{3A}{4m} + \frac{3P}{4m} \right.$$

$$\left. + \frac{3\mathcal{Y}}{m} + \frac{3AP}{m} + \frac{3\mathcal{Y}P}{m} + \frac{3A\mathcal{Y}}{m} \right]$$

$$+ \epsilon \cos 2s \left[\frac{3P}{2m} + \frac{3P\mathcal{Y}}{m} + \frac{3P\mathcal{Y}}{m} + \frac{3P\mathcal{Y}}{m} \right]$$

$$+ \frac{3P}{2m} + \frac{3P\mathcal{Y}}{m} + \frac{3P\mathcal{Y}}{m} + \frac{3P\mathcal{Y}}{m}$$

$$\begin{aligned}
& + ee \cos(2\eta - 2\epsilon) \left\{ \begin{aligned} & + \frac{2}{n} - 2k\mathcal{E} - \mathcal{E}T - \frac{9}{8m} P + \frac{9}{n} \mathcal{C} \\ & + \frac{3AS}{n} + \frac{3\mathcal{E}S}{n} + \frac{3A}{n} \mathcal{C} \end{aligned} \right. \\
& + ee \cos(2\eta + 2\epsilon) \left\{ \begin{aligned} & + \frac{2}{n} - 2k\mathcal{Q} - \mathcal{E}V - \frac{9}{8m} P + \frac{9}{n} \mathcal{C} \\ & + \frac{3AS}{n} + \frac{3\mathcal{E}S}{n} + \frac{3A}{n} \mathcal{C} \end{aligned} \right.
\end{aligned}$$

§. 142. Quodsi iam forma pro R dr assumta differentietur, orietur:

$$\begin{aligned}
R = & - 2a\mathcal{E} \sin 2\eta \\
& + e \sin \epsilon \left(-\frac{1}{n} \eta - \frac{(\mathcal{Q} - \mathcal{R})(2kA + \mathcal{E})}{n} \right) \\
& + e \sin(2\eta - \epsilon) \left(-\frac{2\mathcal{E}}{n} + \frac{\mathcal{E}(2kP + \mathcal{E})}{n} - (2a - \frac{1}{n})\mathcal{Q} \right) \\
& + e \sin(2\eta + \epsilon) \left(-\frac{2\mathcal{E}}{n} + \frac{\mathcal{E}(2kP + \mathcal{E})}{n} - (2a + \frac{1}{n})\mathcal{R} \right) \\
& + ee \sin 2\epsilon \left(\frac{1}{n} \eta - \frac{2}{n} \mathcal{C} - \frac{(\mathcal{E} - \mathcal{R})(2kA + \mathcal{E})}{n} \right) \\
& + ee \sin(2\eta - 2\epsilon) \left(\frac{1}{2n} \mathcal{E} - \frac{3}{n} \mathcal{Q} - (2a - \frac{2}{n})\mathcal{E} \right) \\
& + ee \sin(2\eta + 2\epsilon) \left(\frac{1}{2n} \mathcal{E} - \frac{1}{n} \mathcal{R} - (2a + \frac{2}{n})\mathcal{R} \right)
\end{aligned}$$

§. 143. Comparatione ergo infinita habebitur

$$\frac{1}{n} \eta - \frac{(\mathcal{Q} - \mathcal{R})(2kA + \mathcal{E})}{n} = \frac{3(\mathcal{Q} - \mathcal{R})}{2m}$$

$$\begin{aligned}
& \frac{P}{n} + \frac{9}{n} \mathcal{C} \\
& \frac{A}{n} \mathcal{C} \\
& \frac{P}{n} + \frac{9}{n} \mathcal{C} \\
& \frac{\mathcal{C}}{n}
\end{aligned}$$

ita differentietur

$$\begin{aligned}
& \frac{9}{n} \\
& -\frac{1}{n} \mathcal{Q} \\
& \frac{1}{n} \mathcal{R} \\
& \frac{A + \mathcal{E}}{n} \\
& \frac{2}{n} \mathcal{E} \\
& \mathcal{R}
\end{aligned}$$

cur

unde deinceps valores litterarum germanicarum \mathcal{P} , \mathcal{Q} , \mathcal{R} , \mathcal{C} , \mathcal{E} , \mathcal{S} sumus inuestigaturi.

§. 144. Differentietur simili modo quantitas σ , ac ponatur:

$$\begin{aligned}
\frac{d\sigma}{dt} = & -A/\sin 2\eta - P/e \sin \epsilon - Q/e \sin(2\eta - \epsilon) - S/ee \sin 2\epsilon - T'/ee \sin(2\eta - 2\epsilon) \\
& - R'/e \sin(2\eta + \epsilon) - V'/ee \sin(2\eta + 2\epsilon)
\end{aligned}$$

erique

A' = 2a \mathcal{E} , cuius quidem valor iam supra habetur

$$P' = \frac{1}{n} P + \frac{(\mathcal{Q} - \mathcal{R})(2kA + \mathcal{E})}{n}$$

$$Q' = (2a - \frac{1}{n}) Q + \frac{2}{n} A - \frac{A(2kP + \mathcal{E})}{n}$$

$$R' = (2a + \frac{1}{n}) R + \frac{2}{n} A - \frac{A(2kP + \mathcal{E})}{n}$$

$$S' = \frac{2}{n} S - \frac{1}{n} P + \frac{(T - V)(2kA + \mathcal{E})}{n}$$

Q 3

T'

$$TV = (2a - \frac{2}{n})T + \frac{3}{n}Q - \frac{1}{2n}A$$

$$V' = (2a + \frac{2}{n})V + \frac{1}{n}R - \frac{1}{2n}A$$

unde demum differentiando eruitur.

$$\frac{dV}{dt} = e \operatorname{cof} s \left(-\frac{1}{n}P' + \frac{(Q'+R)(2kA+\mathfrak{N})}{m} \right)$$

$$e \operatorname{cof}(2\eta - s) \left(-(2a - \frac{1}{n})Q' - \frac{2}{n}A' + \frac{A'(2kP+\mathfrak{N})}{m} \right)$$

$$e \operatorname{cof}(2\eta + s) \left(-(2a + \frac{1}{n})R' - \frac{2}{n}A' + \frac{A'(2kP+\mathfrak{N})}{m} \right)$$

$$e \operatorname{cof} 2s \left(-\frac{2}{n}S' + \frac{1}{n}P' + \frac{(T'+V')(2kA+\mathfrak{N})}{m} \right)$$

$$e \operatorname{cof}(2\eta - 2s) \left(-(2a - \frac{2}{n})T' + \frac{1}{2n}A' - \frac{3}{n}Q' \right)$$

$$e \operatorname{cof}(2\eta + 2s) \left(-(2a + \frac{2}{n})V' + \frac{1}{2n}A' - \frac{1}{n}R' \right)$$

§. 145. Sequentes ergo aequationes resolvenda

occurrent

$$-\frac{1}{2} - 2k\mathfrak{N} - 6P - \frac{9A}{4m} + \frac{(3\mathfrak{N}+3A)(Q'R)}{m} + \frac{(3\mathfrak{N}+3A)(Q'R)}{m} =$$

$$-\frac{1}{n}P' + \frac{(Q'+R)(2kA+\mathfrak{N})}{m}$$

$$-\frac{1}{2} - 2k\mathfrak{N} - 6Q - \frac{3A}{4m} + \frac{3P}{4m} + \frac{(3\mathfrak{N}+3A)\mathfrak{N}}{m} + \frac{(3\mathfrak{N}+3A)P}{m} =$$

$$-(2a - \frac{1}{n})Q' - \frac{2}{n}A' + \frac{A'(2kP+\mathfrak{N})}{m}$$

$$-\frac{1}{2} - 2k\mathfrak{N} - 6R - \frac{3A}{4m} + \frac{3P}{4m} + \frac{(3\mathfrak{N}+3A)\mathfrak{N}}{m} + \frac{(3\mathfrak{N}+3A)P}{m} =$$

$$-(2a + \frac{1}{n})R' - \frac{2}{n}A' + \frac{A'(2kP+\mathfrak{N})}{m}$$

$$+\frac{1}{2} - 2k\mathfrak{N} - 6S - \frac{3P}{4m} + \frac{3Q}{4m} + \frac{6P}{4m} + \frac{3P}{4m} =$$

$$-\frac{2}{n}S' + \frac{1}{n}P' + \frac{(T'+V')(2kA+\mathfrak{N})}{m}$$

$$+\frac{1}{2} - 2k\mathfrak{N} - 6T - \frac{9P}{8m} + \frac{(3\mathfrak{N}+3A)\mathfrak{N}}{m} + \frac{(3\mathfrak{N}+3A)S}{m} =$$

$$-(2a - \frac{2}{n})T' + \frac{1}{2n}A' - \frac{3}{n}Q'$$

$$+\frac{1}{2} - 2k\mathfrak{N} - 6V - \frac{9P}{8m} + \frac{(3\mathfrak{N}+3A)\mathfrak{N}}{m} + \frac{(3\mathfrak{N}+3A)S}{m} =$$

$$-(2a + \frac{2}{n})V' + \frac{1}{2n}A' - \frac{1}{n}R'$$

Neglectis primo terminis minimis, qui adhuc funt incogniti, reperitur:

$Q \equiv +4,3238$	$Q \equiv +2,5714$	$Q' \equiv 4,40924$
$R \equiv +1,2210$	$R \equiv +2,0571$	$R' \equiv 3,79793$
$\mathfrak{N} \equiv -0,0313$	$P \equiv -1,4807$	$P' \equiv -0,12185$

§. 146. Ex his autem accuratius ita definiuntur

ut sit:

$Q \equiv +1,33859$	$Q \equiv +1,33859$	$Q' \equiv 0,126649$
$R \equiv +1,23468$	$R \equiv +1,23468$	$R' \equiv 0,091545$
$Q \equiv +2,60087$	$Q \equiv +2,60087$	$Q' \equiv 0,415119$
$R \equiv +2,00590$	$R \equiv +2,00590$	$R' \equiv 0,302308$
$Q' \equiv 4,44801$	$Q' \equiv 4,44801$	$R' \equiv 3,6758E$

et

et $\mathcal{P} = -0,04010 \dots$ $L\mathcal{P} = 8,603133$
 $P = -1,46488 \dots$ $L-P = 0,165801$
 $P' = -0,12222$

Deinde reperitur

$\mathcal{C} = +0,03911 \dots$ $L\mathcal{C} = 8,592230$
 $S = +0,60882 \dots$ $L S = 9,784486$
 $\mathcal{E} = -0,85267 \dots$ $L\mathcal{E} = 9,930827$
 $\mathcal{R} = -0,61125 \dots$ $L\mathcal{R} = 9,793266$
 $T = -2,60380 \dots$ $L-T = 0,465615$
 $V = -1,02720 \dots$ $L-V = 0,011672$

Pro sequentibus vero calculis est

$\frac{2kP+\mathcal{P}}{m} = -0,017051 \dots$ $L\frac{(2kP+\mathcal{P})}{m} = 8,231755$
 $\frac{2kQ+\mathcal{Q}}{m} = +0,037487 \dots$ $L\frac{2kQ+\mathcal{Q}}{m} = 8,573878$
 $\frac{2kR+\mathcal{R}}{m} = +0,030062 \dots$ $L\frac{2kR+\mathcal{R}}{m} = 8,478023$

§. 147. Hinc igitur pro distantia lunae a terra cur-
 tata $s = \frac{(1-k^2)^{2m}}{1-k \cos^2 s}$ pars quantitas s ab excentricitate
 orbitae solaris tantum pendens erit

$\log. \text{coeff.}$
 $\frac{m}{m} = \text{Præced.} = 0,008336e \text{ coef } s$ $7,920985$
 $+ 0,014801e \text{ coef}(2\eta-s)$ $8,170303$
 $+ 0,011415e \text{ coef}(2\eta+s)$ $8,057492$
 $+ 0,00364e \text{ coef } 2s$ $7,539670$
 $- 0,01482e \text{ coef}(2\eta-2s)$ $8,170799$
 $- 0,00584e \text{ coef}(2\eta+2s)$ $7,766856$
 Deinde

Deinde vero erit

$\log. \text{coeff.}$
 $\frac{d\mathcal{Q}}{ds} = \text{Præc.} + 0,017041e \text{ coef } s$ $8,231755$
 $- 0,037487e \text{ coef}(2\eta-s)$ $8,573878$
 $- 0,030062e \text{ coef}(2\eta+s)$ $8,478023$
 $- 0,00722e \text{ coef } 2s$ $7,938166$
 $+ 0,03470e \text{ coef}(2\eta-2s)$ $8,540319$
 $+ 0,01533e \text{ coef}(2\eta+2s)$ $8,185614$

§. 148. Ponatur nunc integrale

$\mathcal{Q} = \text{Præc.} + \mathcal{P}' \sin 2\eta + \mathcal{Q}' e \sin s + \mathcal{R}' e \sin(2\eta+s)$
 $+ \mathcal{E}' e \sin 2s + \mathcal{E}' e \sin(2\eta-2s)$
 $+ \mathcal{R}' e \sin(2\eta+2s)$

erit differentiatione peracta:

$+ 0,017051 = \frac{1}{m} \mathcal{P}' - \frac{(\mathcal{Q}' + \mathcal{R}') (2kA + \mathcal{P})}{m}$
 $- 0,037489 = \left(2a - \frac{1}{m}\right) \mathcal{Q}' + \frac{2}{m} \mathcal{R}' - \frac{\mathcal{R}'(2kP + \mathcal{P})}{m}$
 $- 0,030062 = \left(2a + \frac{1}{m}\right) \mathcal{R}' + \frac{2}{m} \mathcal{Q}' - \frac{\mathcal{Q}'(2kP + \mathcal{P})}{m}$
 $- 0,00722 = \frac{2}{m} \mathcal{E}' - \frac{1}{m} \mathcal{P}' - \frac{(\mathcal{E}' + \mathcal{R}') (2kA + \mathcal{P})}{m}$
 $+ 0,03470 = \left(2a - \frac{2}{m}\right) \mathcal{E}' - \frac{1}{2m} \mathcal{R}' + \frac{3}{m} \mathcal{Q}'$
 $+ 0,01533 = \left(2a + \frac{2}{m}\right) \mathcal{R}' - \frac{1}{2m} \mathcal{Q}' + \frac{1}{m} \mathcal{P}'$
 R $\log. \text{coeff.}$
 freque

hæcque his valoribus determinatis

	log. coeff.
$\Phi = \text{Præc.} + 0,236034 \epsilon \sin s$	9,372974
— $0,021889 \epsilon \sin(2s - s)$	8,340237
— $0,016368 \epsilon \sin(2s + s)$	8,214002
+ $0,06615 \epsilon \epsilon \sin 2s$	8,820508
+ $0,02332 \epsilon \epsilon \sin(2s - 2s)$	8,367825
+ $0,00840 \epsilon \epsilon \sin(2s + 2s)$	7,924429

§. 149. Reducamus has inæqualitates etiam ad minuta secunda; ponendo excentricitatem orbitæ solaris $\epsilon = 0,01680$, atque habebimus

	log. coeff.
$\Phi = \text{Præc.} + 817'' , 9 \sin s$	2,912708
— $75, 8 \sin(2s - s)$	1,879971
— $56, 7 \sin(2s + s)$	1,753736
+ $3, 8 \sin 2s$	0,585551
+ $1, 4 \sin(2s - 2s)$	0,132868
+ $0, 5 \sin(2s + 2s)$	9,689472

Denotat hic s anomaliam veram solis; vnde patet eam Lunæ inæqualitatem, quæ finni huius anomalie est proportionalis, admodum esse notabilem, dum ad $13', 38''$ exurgit. Tabulæ autem Astronomicæ, vbi hæc inæqualitas æquatio solaris nominatur, eam multo minorem faciunt, cuius rei causam investigari adhuc con-
veniet.

§. 150.

§. 150. Quodsi enim litteram η' accuratius defini-
re velimus, habebimus has formulas resolvendas:

$$- \frac{1}{n} \eta' + \frac{\eta'(2\pi Q + \Omega)}{n} - \frac{\eta'(2\pi R + S)}{n} - \frac{(\Omega - S)(2\pi A + \mathcal{M})}{n} - \frac{3(QR)}{n}$$

$$P' = \frac{1}{n} P - \frac{A(2\pi Q + \Omega)}{n} + \frac{A(2\pi R + S)}{n} + \frac{(Q - R)(2\pi A + \mathcal{M})}{n}$$

$$- \frac{1}{2} - 2\pi \eta' - \frac{6P - 9A}{4n} + \frac{(2\eta'BA)(\Omega + S)}{n} + \frac{(3\eta'BA)(QR)}{n} =$$

$$- \frac{1}{n} P' + \frac{A(2\pi Q + \Omega)}{n} + \frac{A(2\pi R + S)}{n} + \frac{(QR)(2\pi A + \mathcal{M})}{n}$$

vnde elicimus:

$$\eta' = - 0,1183; P = - 1,1356; \frac{P}{n} = - 0,0064$$

atque $\frac{2\pi P + \mathcal{M}}{n} = - 0,01376$, ferrique iam oportet

$$+ 0,01376 = \frac{1}{n} \eta' - \frac{\eta'(2\pi Q + \Omega)}{n} - \frac{\eta'(2\pi R + S)}{n} - \frac{(\Omega + S)(2\pi A + \mathcal{M})}{n}$$

vnde oritur $\eta' = + 0,201385$. Quare accuratius habemus

$\Phi = \text{Præc.} - 0,006400 \epsilon \cos s$	7,806180
$\frac{d\Phi}{ds} = \text{Præc.} + 0,013760 \epsilon \cos s$	8,138618
$\Phi = \text{Præc.} + 0,201385 \epsilon \sin s$	9,304026

$\Phi = \text{Præc.} + 201'' , 1 \sin s$ | 2,845780
Ergo æquatio finni Anguli ϵ proportionalis tantum
est $11', 41''$.

CAPUT X.

INVESTIGATIO INAEQUALITATVM LUNAE AB VTIUSQUE ORBITAE EXCENTRICITATE SIMUL PENDENTIUM.

§ 151.

Quoniam praevidemus inaequalitates huius generis, quae altiores litterarum k et e potestates simul complectuntur, minimas esse fumras, alios terminos non servabimur, nisi qui producto simplici ek sint affecti. Habebimus ergo

$$\frac{ds}{dt} = \frac{ds}{dt} = \frac{1}{n} + \frac{2}{n} k \cos r - \frac{2}{n} e \cos s - \frac{2}{n} ek \cos(r-s) - \frac{2}{n} ek \cos(r+s)$$

Cum igitur ad hanc investigationem opus non sit illis terminis ex praecedentibus, qui vel per k^2 vel per e^2 erant affecti, quia litterae alphabeti descendere incipiunt, litteris S, T et sequentibus denuo utemur; quare cavendum, ne istae litterae cum ante adhibitis confundantur.

§. 152. Assumtis ergo ex terminis iam ante definitis, iis qui in eos, quos iam investigamus, vim exerunt, ponamus

$$\begin{aligned} R &= \mathcal{M} \cos(2\eta) + \mathcal{E} k \cos r + \mathcal{D} k \cos(2\eta-r) + \mathcal{H} e \cos s + \mathcal{Q} e \cos(2\eta-s) \\ &\quad + \mathcal{G} k \cos(2\eta+r) + \mathcal{H} e \cos(2\eta+r) \\ &\quad + \mathcal{E} k \cos(r-s) + \mathcal{M} e \cos(2\eta-r+s) + \mathcal{H} e \cos(2\eta-r-s) \\ &\quad + \mathcal{E} k \cos(r+s) + \mathcal{H} e \cos(2\eta+r+s) + \mathcal{Q} e \cos(2\eta+r-s) \end{aligned}$$

INAE
TE

inertis, simul s ter-
ici ek
s)
s)
c illis
er e^2
iunt,
e ca-
nfun-
def-
exfe-
2\eta-s)
2\eta+s)
r-s)
r+s)
v =

$$\begin{aligned} v = & A \cos 2\eta \dots + D k \cos(2r-s) + P e \cos s + Q e \cos(2\eta-s) \\ & + E k \cos(2\eta+r) + R e \cos(2\eta+s) \\ & + S e k \cos(r-s) + V e k \cos(2\eta-r+s) + Y e k \cos(2\eta-r-s) \\ & + T e k \cos(r+s) + X e k \cos(2\eta+r-s) + Z e k \cos(2\eta+r+s) \end{aligned}$$

etique ex praecedentibus

$\mathcal{M} = -0,81033$;	$L\mathcal{M} = 9,908662$
$\mathcal{E} = -0,64383$;	$L\mathcal{E} = 9,808771$
$\mathcal{D} = -3,59362$;	$L\mathcal{D} = 0,555531$
$\mathcal{G} = -1,08732$;	$L\mathcal{G} = 0,036358$
$\mathcal{H} = -0,11830$;	$L\mathcal{H} = 9,072985$
$\mathcal{Q} = +1,33859$;	$L\mathcal{Q} = 0,126649$
$\mathcal{M} = +1,23468$;	$L\mathcal{M} = 0,091545$
$A = -1,31773$;	$L A = 0,119826$
$D = +33,6600$;	$L D = 1,527113$
$E = -0,5785$;	$L E = 9,762341$
$P = -1,1356$;	$L P = 0,055225$
$Q = +2,60987$;	$L Q = 0,415119$
$R = +2,00590$;	$L R = 0,302308$

§. 153. Reliqui vero valores hinc derivati, quibus opus habemus, sunt:

$A' = -2,47576$;	$L A' = 0,393708$
$C = -0,13847$;	$L C = 9,141356$
$D' = 29,39153$;	$L D' = 1,468222$
$E' = -1,47347$;	$L E' = 0,168341$
$P' = -0,0260$;	$L P' = 8,414073$
$Q' = 4,40924$;	$L Q' = 0,644363$
$R' = 3,79793$;	$L R' = 0,579548$

- $\mathcal{M}' = +0,01036$; $l \mathcal{M}' = 8,015347$
- $\mathcal{C}' = +0,01015$; $l \mathcal{C}' = 8,006295$
- $\mathcal{D}' = -0,42023$; $l -\mathcal{D}' = 9,623483$
- $\mathcal{E}' = +0,00499$; $l -\mathcal{E}' = 7,698261$
- $\mathcal{M}'' = +0,20138$; $l \mathcal{M}'' = 9,304016$
- $\mathcal{D}'' = -0,02189$; $l -\mathcal{D}'' = 8,340237$
- $\mathcal{M}''' = -0,01637$; $l -\mathcal{M}''' = 8,214002$

Sit brevitatis gratia

- $\mathcal{A}' = \frac{2\mathcal{K}\mathcal{A} + \mathcal{M}'}{m} = -0,019744$; $l \frac{(2\mathcal{K}\mathcal{A} + \mathcal{M}')}{m} = 8,295442$
- $\mathcal{D}' = \frac{2\mathcal{K}\mathcal{D} + \mathcal{D}'}{m} = +0,366111$; $l \frac{2\mathcal{K}\mathcal{D} + \mathcal{D}'}{m} = 9,563604$
- $\mathcal{E}' = \frac{2\mathcal{K}\mathcal{E} + \mathcal{E}'}{m} = -0,01283$; $l \frac{(2\mathcal{K}\mathcal{E} + \mathcal{E}')}{m} = 8,108292$
- $\mathcal{P}' = \frac{2\mathcal{K}\mathcal{P} + \mathcal{P}'}{m} = -0,01376$; $l \frac{(2\mathcal{K}\mathcal{P} + \mathcal{P}')}{m} = 8,138618$
- $\mathcal{Q}' = \frac{2\mathcal{K}\mathcal{Q} + \mathcal{Q}'}{m} = +0,03749$; $l \frac{2\mathcal{K}\mathcal{Q} + \mathcal{Q}'}{m} = 8,573878$
- $\mathcal{R}' = \frac{2\mathcal{K}\mathcal{R} + \mathcal{R}'}{m} = +0,03006$; $l \frac{2\mathcal{K}\mathcal{R} + \mathcal{R}'}{m} = 8,478023$

§. 154. Si simili modo vterius ponatur:

- $\mathcal{S} = \frac{2\mathcal{K}\mathcal{S} + \mathcal{S}'}{2m}$; $\mathcal{P}' = \frac{2\mathcal{K}\mathcal{T} + \mathcal{S}'}{m}$; $\mathcal{V}' = \frac{2\mathcal{K}\mathcal{V} + \mathcal{S}'}{m}$;
- $\mathcal{X}' = \frac{2\mathcal{K}\mathcal{X} + \mathcal{X}'}{m}$; $\mathcal{Y}' = \frac{2\mathcal{K}\mathcal{Y} + \mathcal{Y}'}{m}$; $\mathcal{Z}' = \frac{2\mathcal{K}\mathcal{Z} + \mathcal{Z}'}{m}$

habe-

habebimus :

$$\frac{d\mathcal{Q}}{d\mathcal{P}} = \text{Præc.} - \frac{d' \text{cf}(2\eta - s)}{m} - \frac{\mathcal{E}'}{m} k \text{cf} - \frac{d' k \text{cf}(2\eta - s)}{m} - \mathcal{P}' e \text{cf} - \mathcal{Q}' e \text{cf}(2\eta - s)$$

$$- \mathcal{P}' e k \text{cf}(r - s) - \mathcal{V}' e k \text{cf}(2\eta - r + s) - \mathcal{Y}' e k \text{cf}(2\eta - r - s)$$

$$- \mathcal{M}' e k \text{cf}(r + s) - \mathcal{M}'' e k \text{cf}(2\eta + r - s) - \mathcal{M}''' e k \text{cf}(2\eta + r + s)$$

atque posito $\frac{2}{m} + \frac{\mathcal{E}'}{m} = \mathcal{P}' = 0,147197$; $l \mathcal{P}' = 9,167900$

$$\frac{d\mathcal{Q}}{d\mathcal{P}} = a - \frac{d' \text{cf}(2\eta - s)}{m} - \frac{d' k \text{cf}(2\eta - s)}{m} + \left(\frac{2}{m} - \mathcal{P}'\right) e \text{cf} - \mathcal{Q}' e \text{cf}(2\eta - s)$$

$$+ \left(\frac{2}{m} - \mathcal{P}'\right) e k \text{cf}(r - s) - \mathcal{V}' e k \text{cf}(2\eta - r + s) - \mathcal{Y}' e k \text{cf}(2\eta - r - s)$$

$$+ \left(\frac{2}{m} - \mathcal{P}'\right) e k \text{cf}(r + s) - \mathcal{M}' e k \text{cf}(2\eta + r - s) - \mathcal{M}'' e k \text{cf}(2\eta + r + s)$$

vbi cum sit $\frac{2}{m} = 0,150876$, erit $\frac{2}{m} - \mathcal{P}' = 0,16464$

§. 155. Nunc termini coefficiente $e k$ affici, qui in formis R = et $\frac{d' d''}{d' p^2}$ insunt, colligantur: eritque

$$R = e k \text{fn}(r - s) \left(+ \frac{3}{2m} Y - \frac{3}{2m} X - \frac{3\mathcal{Q}'}{m} + \frac{3\mathcal{R}'}{m} - \frac{9\mathcal{D}'}{4m} + \frac{9\mathcal{E}'}{4m} \right)$$

$$e k \text{fn}(r + s) \left(+ \frac{3}{2m} Y - \frac{3}{2m} X - \frac{3\mathcal{R}'}{m} + \frac{3\mathcal{Q}'}{m} + \frac{9\mathcal{E}'}{4m} - \frac{9\mathcal{D}'}{4m} \right)$$

$$e k \text{fn}(2\eta - r + s) \left(- \frac{2}{m} + \frac{3}{2m} S + \frac{3\mathcal{P}'}{m} \right)$$

$$e k \text{fn}(2\eta + r - s) \left(- \frac{2}{m} + \frac{3}{2m} S + \frac{3\mathcal{P}'}{m} \right)$$

436

CAPUT X.

$$ck \sin(2\eta - r - s) \left(-\frac{1}{2} + \frac{3}{2m} T + \frac{3P}{2m} \right)$$

$$ck \sin(2\eta + r + s) \left(-\frac{1}{2} + \frac{3}{2m} T + \frac{3P}{2m} \right)$$

or $\frac{ddv}{d\eta^2}$

$$\left[\begin{aligned} & -3-2k\mathcal{E}-6S+\frac{1}{2}bP + \frac{3V}{4m} + \frac{3X}{4m} + \frac{3Q}{2m} + \frac{3R}{2m} \\ & -\frac{9D}{8m} - \frac{9E}{8m} + \frac{9\mathcal{G}}{m} + \frac{9\mathcal{H}}{m} + \frac{9\mathcal{I}}{m} + \frac{9\mathcal{J}}{m} + \frac{9\mathcal{K}}{m} \\ & + \frac{3AV}{m} + \frac{3AX}{m} + \frac{3DQ}{m} + \frac{3ER}{m} - \frac{3AQ}{2m} - \frac{3AR}{2m} \\ & + \frac{3\mathcal{M}V}{m} + \frac{3A\mathcal{G}}{m} + \frac{3\mathcal{M}X}{m} + \frac{3A\mathcal{H}}{m} + \frac{3\mathcal{C}P}{m} \\ & + \frac{3DQ}{m} + \frac{3D\mathcal{I}}{m} + \frac{3\mathcal{C}R}{m} + \frac{3\mathcal{E}R}{m} \end{aligned} \right]$$

ck cos(r-s)

$$\left[\begin{aligned} & -3-2k\mathcal{E}-6T+\frac{1}{2}bP + \frac{3Y}{4m} - \frac{3Z}{4m} + \frac{3Q}{2m} + \frac{3R}{2m} \\ & -\frac{9D}{8m} - \frac{9E}{8m} + \frac{9\mathcal{G}}{m} + \frac{9\mathcal{H}}{m} + \frac{9\mathcal{I}}{m} + \frac{9\mathcal{J}}{m} + \frac{9\mathcal{K}}{m} \\ & + \frac{3AY}{m} + \frac{3AZ}{m} + \frac{3DR}{m} + \frac{3EQ}{2m} - \frac{3AQ}{2m} - \frac{3AR}{2m} \\ & + \frac{3\mathcal{M}Y}{m} + \frac{3A\mathcal{H}}{m} + \frac{3\mathcal{M}Z}{m} + \frac{3A\mathcal{I}}{m} + \frac{3\mathcal{C}P}{m} \\ & + \frac{3EQ}{m} + \frac{3E\mathcal{I}}{m} + \frac{3\mathcal{C}R}{m} + \frac{3D\mathcal{R}}{m} \end{aligned} \right]$$

ck cos(r+s)

ck

CAPUT X.

137

$$ck \cos(2\eta - r - s) \left[\begin{aligned} & -\frac{1}{2}-2k\mathcal{E}-6V+\frac{1}{2}bR + \frac{3S}{4m} - \frac{3D}{4m} + \frac{3P}{2m} \\ & + \frac{9\mathcal{C}}{m} + \frac{9\mathcal{D}}{m} + \frac{9\mathcal{E}}{m} + \frac{3AS}{m} + \frac{3DP}{m} - \frac{3AP}{2m} \\ & + \frac{3\mathcal{M}S}{m} + \frac{3A\mathcal{C}}{m} + \frac{3\mathcal{C}R}{m} + \frac{3DP}{m} + \frac{3D\mathcal{H}}{m} \end{aligned} \right]$$

$$ck \cos(2\eta + r - s) \left[\begin{aligned} & -\frac{1}{2}-2k\mathcal{E}-6X+\frac{1}{2}bQ + \frac{3S}{4m} - \frac{3E}{4m} + \frac{3P}{2m} \\ & + \frac{9\mathcal{C}}{m} + \frac{9\mathcal{D}}{m} + \frac{9\mathcal{E}}{m} + \frac{3AS}{m} + \frac{3EP}{m} - \frac{3AP}{2m} \\ & + \frac{3\mathcal{M}S}{m} + \frac{3A\mathcal{C}}{m} + \frac{3\mathcal{C}Q}{m} + \frac{3\mathcal{C}P}{m} + \frac{3E\mathcal{H}}{m} \end{aligned} \right]$$

ck cos(2\eta+r+s)

$$ck \cos(2\eta - r - s) \left[\begin{aligned} & -\frac{1}{2}-2k\mathcal{E}-6Y+\frac{1}{2}bQ + \frac{3T}{4m} - \frac{3D}{4m} + \frac{3P}{2m} \\ & + \frac{9\mathcal{C}}{m} + \frac{9\mathcal{D}}{m} + \frac{9\mathcal{E}}{m} + \frac{3AT}{m} + \frac{3DP}{m} - \frac{3AP}{2m} \\ & + \frac{3\mathcal{M}T}{m} + \frac{3A\mathcal{C}}{m} + \frac{3\mathcal{C}Q}{m} + \frac{3DP}{m} + \frac{3D\mathcal{H}}{m} \end{aligned} \right]$$

ck cos(2\eta+r+s)

$$ck \cos(2\eta + r - s) \left[\begin{aligned} & -\frac{1}{2}-2k\mathcal{E}-6Z+\frac{1}{2}bR + \frac{3T}{4m} - \frac{3E}{4m} + \frac{3P}{2m} \\ & + \frac{9\mathcal{C}}{m} + \frac{9\mathcal{D}}{m} + \frac{9\mathcal{E}}{m} + \frac{3AT}{m} + \frac{3EP}{m} - \frac{3AP}{2m} \\ & + \frac{3\mathcal{M}T}{m} + \frac{3A\mathcal{C}}{m} + \frac{3\mathcal{C}R}{m} + \frac{3\mathcal{C}P}{m} + \frac{3E\mathcal{H}}{m} \end{aligned} \right]$$

ck cos(2\eta+r+s)

S

§. 156. Queramus ergo quoque ex formula as-
sumta $\int R dx$ differentiale, quod erit

$$R = \frac{\dots}{\dots}$$

$$\begin{aligned} &ek\sin(r-s)(+\mathfrak{R}g^d-\mathfrak{R}g^d-\mathfrak{Q}g^d+\mathfrak{E}g^d+\frac{1}{n}\mathfrak{H}+\mathfrak{Q}d^d-\mathfrak{R}d^d-(1-\frac{1}{n})\mathfrak{E}-\mathfrak{R}g^d+\mathfrak{R}d^d) \\ &ek\sin(r+s)(+\mathfrak{R}g^d-\mathfrak{R}g^d-\mathfrak{Q}g^d+\mathfrak{E}g^d-\frac{1}{n}\mathfrak{H}-\mathfrak{Q}d^d+\mathfrak{R}d^d-(1+\frac{1}{n})\mathfrak{E}-\mathfrak{R}g^d+\mathfrak{R}d^d) \\ &ek\sin(2q+r+s)(-\mathfrak{R}(\frac{2}{n}-s^d)-\mathfrak{D}(\frac{2}{n}-p^d)+\mathfrak{R}d^d-\frac{1}{n}\mathfrak{R}-(2a-1+\frac{1}{n})\mathfrak{R}) \\ &ek\sin(2q+r-s)(-\mathfrak{R}(\frac{2}{n}-s^d)-\mathfrak{D}(\frac{2}{n}-p^d)+\mathfrak{Q}d^d+\frac{1}{n}\mathfrak{Q}-(2a+1-\frac{1}{n})\mathfrak{R}) \\ &ek\sin(2q+r+s)(-\mathfrak{R}(\frac{2}{n}-s^d)-\mathfrak{D}(\frac{2}{n}-p^d)+\mathfrak{Q}d^d+\frac{1}{n}\mathfrak{Q}-(2a-1-\frac{1}{n})\mathfrak{R}) \\ &ek\sin(2q+r-s)(-\mathfrak{R}(\frac{2}{n}-s^d)-\mathfrak{D}(\frac{2}{n}-p^d)+\mathfrak{R}d^d-\frac{1}{n}\mathfrak{R}-(2a+1+\frac{1}{n})\mathfrak{R}) \end{aligned}$$

§. 157. Ponatur ex differentiatione formae v;

$$\begin{aligned} S' &= (1-\frac{1}{n})S-A(v^d-x^d)+Dg^d-Es^d-\frac{1}{n}P-Qd^d+Rd^d+(V-X)g^d \\ T' &= (1+\frac{1}{n})T-A(v^d-x^d)+Dg^d-Es^d+\frac{1}{n}P+Qd^d-Rd^d+(Y-Z)g^d \\ V' &= (2a-1+\frac{1}{n})V+A(\frac{2}{n}-s^d)+D(\frac{2}{n}-p^d)-Rd^d+\frac{1}{n}R \\ X' &= (2a+1-\frac{1}{n})X+A(\frac{2}{n}-s^d)+E(\frac{2}{n}-p^d)-Qd^d-\frac{1}{n}Q \\ Y' &= (2a-1-\frac{1}{n})Y+A(\frac{2}{n}-s^d)+D(\frac{2}{n}-p^d)-Qd^d-\frac{1}{n}Q \\ Z' &= (2a+1+\frac{1}{n})Z+A(\frac{2}{n}-s^d)+E(\frac{2}{n}-p^d)-Rd^d+\frac{1}{n}R \end{aligned}$$

VI

ha as-

$$\begin{aligned} &+ \mathfrak{R}g^d \\ &+ \mathfrak{R}d^d \\ &+ \mathfrak{R}d^d \end{aligned}$$

§. 158. Haec iam forma denuo differentiata dabit
 $\frac{d^2v}{dx^2} = \text{Praec.}$

$$\begin{aligned} &+ ek\cos(r-s)(+A(v^d+x^d)+Dg^d+Eg^d-\frac{1}{n}P+Qd^d+Rd^d-(1-\frac{1}{n})S'+(V+X)g^d \\ &+ ek\cos(r+s)(+A(v^d+x^d)+Dg^d+Eg^d-\frac{1}{n}P+Qd^d+Rd^d-(1+\frac{1}{n})T'+(Y+Z)g^d \\ &+ ek\cos(2q+r+s)(-A(\frac{2}{n}-s^d)-D(\frac{2}{n}-p^d)+\frac{1}{n}R'-(2a-1+\frac{1}{n})V') \\ &+ ek\cos(2q+r-s)(-A(\frac{2}{n}-s^d)-D(\frac{2}{n}-p^d)+\frac{1}{n}R'-(2a-1-\frac{1}{n})V') \\ &+ ek\cos(2q+r+s)(-A(\frac{2}{n}-s^d)-E(\frac{2}{n}-p^d)+Qd^d+\frac{1}{n}Q'-(2a+1+\frac{1}{n})Y') \\ &+ ek\cos(2q+r-s)(-A(\frac{2}{n}-s^d)-E(\frac{2}{n}-p^d)+Qd^d+\frac{1}{n}Q'-(2a+1-\frac{1}{n})Y') \\ &+ ek\cos(2q+r+s)(-A(\frac{2}{n}-s^d)-D(\frac{2}{n}-p^d)+\frac{1}{n}R'-(2a+1+\frac{1}{n})Z') \\ &+ ek\cos(2q+r-s)(-A(\frac{2}{n}-s^d)-D(\frac{2}{n}-p^d)+\frac{1}{n}R'-(2a+1-\frac{1}{n})Z') \end{aligned}$$

§. 159. Priores autem expressiones, si litterarum cogni-
tarum valores substituamur, sequens modo prodibunt.

$$\begin{aligned} R &= ek\sin(r-s) (-0,44856 + 0,00854(V-X)) \\ &ek\sin(r+s) (-0,42826 + 0,00854(Y-Z)) \\ &ek\sin(2q+r+s) (-4,51938 + 9,00854S) \\ &ek\sin(2q+r-s) (-4,51938 + 0,00854S) \\ &ek\sin(2q+r+s) (-4,51939 + 0,00854T) \\ &ek\sin(2q+r-s) (-4,51938 + 0,00854T) \end{aligned}$$

§. 160.

§. 160. Altera vero forma pro $\frac{dho}{dx^2}$ fit

$$\begin{aligned} & \frac{dho}{dx^2} = \\ & ekcf(r-s)(-2,83505-2k\mathcal{E}-6S-0,02711(\mathcal{R}+\mathcal{E})-0,03207(V+X)) \\ & ekcf(r+s)(-3,21669-2k\mathcal{E}-6T-2,02711(\mathcal{R}+\mathcal{E})-0,03207(Y+Z)) \\ & ekcf(2r-r+s)(-2,26927-2k\mathcal{E}-6V-0,02711\mathcal{E}-0,03207S) \\ & ekcf(2r+r-s)(-0,53441-2k\mathcal{E}-6X-0,02711\mathcal{E}-0,03207S) \\ & ekcf(2r-r-s)(-1,36456-2k\mathcal{E}-6Y-0,02711\mathcal{E}-0,03207T) \\ & ekcf(2r+r+s)(-1,43912-2k\mathcal{E}-6Z-0,02711\mathcal{E}-0,03207T) \end{aligned}$$

§. 161. Deinde firmi modo alterae formulae per differentiationem erunt, sufficientis valoribus cognitis ita se habebunt.

$$\begin{aligned} R & = \\ ekfm(r-s)(+0,59883+\mathcal{R}(v'-x')\cdot(1-\frac{1}{n}))\mathcal{E}+0,01974(\mathcal{R}-\mathcal{E}) \\ ekfm(r+s)(+0,54556+\mathcal{R}(y'-z')\cdot(1+\frac{1}{n}))\mathcal{E}+0,01974(\mathcal{R}-\mathcal{E}) \\ ekfm(2r-r+s)(+0,80251+\mathcal{R}y'-(\frac{2a-1}{n}))\mathcal{R} \\ ekfm(2r+r-s)(+0,59936+\mathcal{R}y'-(\frac{2a+1}{n}))\mathcal{E} \\ ekfm(2r-r-s)(+1,01199+\mathcal{R}y'-(\frac{2a-1}{n}))\mathcal{R} \\ ekfm(2r+r+s)(+0,38988+\mathcal{R}y'-(\frac{2a+1}{n}))\mathcal{E} \end{aligned}$$

Porro

Porro reperiemus sequentes valores

$$\begin{aligned} S & = (1-\frac{1}{n})S-A(v'-x')+0,38693-0,01974(V-X) \\ T & = (1+\frac{1}{n})T-A(y'-z')+0,18018-0,01974(Y-Z) \\ V & = (2a-1+\frac{1}{n})V-Ay'+4,19902 \\ X & = (2a+1-\frac{1}{n})X-Ay'-0,87311 \\ Y & = (2a-1-\frac{1}{n})Y-Ay'+4,76391 \\ Z & = (2a+1+\frac{1}{n})Z-Ay'-0,43800 \end{aligned}$$

ac denique $\frac{dho}{dx^2} =$ Praec.

$$\begin{aligned} & + ekcf(r-s)(-(1-\frac{1}{n})S'+A'(v'+x')+2,62592-0,01974(V+X)) \\ & + ekcf(r+s)(-(1+\frac{1}{n})T'+A'(y'+z')+2,16417-0,01974(Y+Z)) \\ & + ekcof(2r-r+s)(-(2a-1+\frac{1}{n})V'+A'y'-4,19296) \\ & + ekcof(2r+r-s)(-(2a+1-\frac{1}{n})X'+A'y'+1,59777) \\ & + ekcof(2r-r-s)(-(2a-1-\frac{1}{n})Y'+A'y'-3,48384) \\ & + ekcof(2r+r+s)(-(2a+1+\frac{1}{n})Z'+A'y'+0,88865) \end{aligned}$$

Porro

§. 162. Hinc ergo pro determinandis coefficientibus sequentes obinemus aequationes

$$(1 - \frac{1}{n}) \textcircled{2} = 1,04739 - 0,00854(V-X) + \textcircled{3}(v'-x') + 0,01974(\textcircled{3}-\textcircled{2})$$

$$(1 + \frac{1}{n}) \textcircled{2} = 0,97382 - 0,00858(Y-Z) + \textcircled{3}(y'-z') + 0,01974(\textcircled{3}-\textcircled{2})$$

$$(2n-1 + \frac{1}{n}) \textcircled{3} = 5,32189 - 0,00854S + \textcircled{3}S'$$

$$(2n+1 - \frac{1}{n}) \textcircled{3} = 5,11874 - 0,00854S + \textcircled{3}S'$$

$$(2n-1 - \frac{1}{n}) \textcircled{3} = 5,53137 - 0,00854T + \textcircled{3}T'$$

$$(2n+1 + \frac{1}{n}) \textcircled{3} = 4,90926 - 0,00854T + \textcircled{3}T'$$

Deinde

$$+5,46097 = (1 - \frac{1}{n})S' - 2n \textcircled{2} - 5S - 0,02711(\textcircled{3} + \textcircled{3}') - 0,03207(V+X)$$

$$- A'(v'+x') + 0,01974(V'+X')$$

$$+5,38086 = (1 + \frac{1}{n})T' - 2n \textcircled{2} - 5T - 0,02711(\textcircled{3} + \textcircled{3}') - 0,03207(Y+Z)$$

$$- A'(y'+z') + 0,01974(Y'+Z')$$

$$-1,92371 = (2n-1 + \frac{1}{n})V' - 2n \textcircled{3} - 5V - 0,02711 \textcircled{2} + 0,03207S - A'S'$$

$$+2,13218 = (2n+1 - \frac{1}{n})X' - 2n \textcircled{3} - 5X - 0,02711 \textcircled{2} + 0,03207S - A'X'$$

$$-2,11928 = (2n-1 - \frac{1}{n})Y' - 2n \textcircled{3} - 5Y - 0,02711 \textcircled{2} + 0,03207T - A'Y'$$

$$+2,32777 = (2n+1 + \frac{1}{n})Z' - 2n \textcircled{3} - 5Z - 0,02711 \textcircled{2} + 0,03207T - A'Z'$$

§. 163.

icenti.

3-2)

3) 3)

2

2

2

2

F

S

S

0

0

§. 163. Pro ulteriori calculo est

$$1 - \frac{1}{n} = 0,924562 \dots \dots \dots f(1 - \frac{1}{n}) = 9,965935$$

$$1 + \frac{1}{n} = 1,075438 \dots \dots \dots f(1 + \frac{1}{n}) = 0,031570$$

$$2n-1 + \frac{1}{n} = 0,942914 \dots \dots \dots f(2n-1 + \frac{1}{n}) = 9,965935$$

$$2n+1 - \frac{1}{n} = 2,792038 \dots \dots \dots f(2n+1 - \frac{1}{n}) = 0,445915$$

$$2n-1 - \frac{1}{n} = 0,792038 \dots \dots \dots f(2n-1 - \frac{1}{n}) = 9,898747$$

$$2n+1 + \frac{1}{n} = 2,942914 \dots \dots \dots f(2n+1 + \frac{1}{n}) = 0,468775$$

Hinc in aequationibus posterioribus valores litterarum S', T', V' habbittantur, et ob 5 = 1,01591, erit

$$0,16110S = -5,10323 - 2n \textcircled{2} - A'(v'+x') - 0,02711(\textcircled{3} + \textcircled{3}') + 0,01974(V'+X')$$

$$+ 1,21835(v'-x') - 0,03207(V+X) - 0,01825(V-X)$$

$$0,14059T = +5,18709 + 2n \textcircled{2} + A'(y'+z') + 0,02711(\textcircled{3} + \textcircled{3}') - 0,01974(Y'+Z')$$

$$- 1,41710(y'-z') + 0,03207(Y+Z) + 0,02124(Y-Z)$$

$$0,12683V = +6,82591 - 2n \textcircled{3} + 3,718S' - 0,02711 \textcircled{2} - 0,03207S$$

$$6,77934X = +4,56988 + 2n \textcircled{3} - 6,1548S' + 0,02711 \textcircled{2} + 0,03207S$$

$$0,38858Y = +5,89253 - 2n \textcircled{3} + 3,5194S' - 0,02711 \textcircled{2} - 0,03207T$$

$$7,64474Z = +3,61676 + 2n \textcircled{3} - 6,3536S' + 0,02711 \textcircled{2} + 0,03207T$$

§. 164.

53.

§. 164. Commodissime hi coefficients inveniuntur, si primo Q, R, S et V, X, Y, Z proxime quaerantur, quod fiet terminos minimos negligendo:

Q	= + 5,7560	...	/ Q	= 0,760125	
R	= + 1,8334	...	/ R	= 0,263245	
S	= + 6,9837	...	/ S	= 0,844088	
V	= + 1,6643	...	/ V	= 0,221240	
X	= - 37,7650	...	/ X	= 1,577086	
Y	= + 1,2198	...	/ Y	= 0,086293	
Z	= - 21,1040	...	/ Z	= 1,324360	
V'	= + 0,9125	...	/ V'	= 9,960209	
X'	= - 29,7170	...	/ X'	= - 0,398	
Y'	= + 2,5326	...	/ Y'	= + 0,024	
Z'	= - 11,9510	...	/ Z'	= - 0,201	
	Z	= + 2,2472	...	Z	= + 0,020

§. 165. Hinc autem valores pro V' et Y' tam sane magni, ut vicissim post inventas litteras S, T nihil valores modo erutos efficiant, vnde necesse erit reductionem harum aequationum ordinario modo invenire. Reperitur ergo

Q	= 5,7560	...	Q	= 0,0050
R	= 1,8333	...	R	= 0,0016
S	= 6,9837	...	S	= 0,0058
V	= 1,6643	...	V	= 0,0016
X	= 11,6155	...	X	= 0,0100
Y	= 3,6996	...	Y	= 0,0033
Z	= 14,0930	...	Z	= 0,0118
V'	= 3,3586	...	V'	= 0,0032

qui

qui valores substitui dant:

V	= - 37,7647	...	V	= 0,0031
X	= + 1,2198	...	X	= 0,0016
Y	= - 21,1038	...	Y	= 0,0121
Z	= + 0,9125	...	Z	= 0,0016
V'	= - 76,2085	...	V'	= 0,0064
X'	= + 2,4615	...	X'	= 0,0033
Y'	= - 42,5870	...	Y'	= 0,0244
Z'	= + 1,8413	...	Z'	= 0,0033

§. 166. Hinc porro valores derivati erunt

V'	= - 29,7167	...	V'	= 0,0094
X'	= + 2,5326	...	X'	= 0,0029
Y'	= - 11,9511	...	Y'	= 0,0161
Z'	= + 2,2472	...	Z'	= 0,0028
V''	= - 0,4009	...	V''	= 0,0001
X''	= + 0,0244	...	X''	= 0,0001
Y''	= - 0,2026	...	Y''	= 0,0001
Z''	= + 0,0200	...	Z''	= 0,0001

ac porro

Q	= 3,9227	...	Q	= 0,0034
R	= 5,3194	...	R	= 0,0042
S	= 38,9845	...	S	= 0,0047
V	= - 22,0163	...	V	= 0,0137
X	= 7,5893	...	X	= 0,0066
Y	= 8,6480	...	Y	= 0,0074
Z	= 36,5449	...	Z	= 0,0015
V'	= - 20,1913	...	V'	= 0,0105

T

V'+X'

$V+X' = -27,1841 + 0,3706S + 0,0123\text{C}$
 $Y+Z' = -9,7039 + 0,1194T + 0,0189\text{C}$
 $v'-s' = -0,4253 + 0,0044S + 0,0001\text{C}$
 $y'-z' = -0,2226 + 0,0015T + 0,0001\text{C}$
 $v'+s' = -0,3765 + 0,0044S + 0,0001\text{C}$
 $y'+z' = -0,1826 + 0,0015T + 0,0001\text{C}$

§. 167. His valoribus satisfactis reperitur

$$\left(1 - \frac{1}{n}\right) \text{C} = +1,8024 - 0,0070S$$

$$\left(1 + \frac{1}{n}\right) \text{C} = +1,4453 - 0,0027T$$

unde concluditur

$\text{C} = 1,9495 - 0,90075S$ $2\text{C} = 3,9340 - 0,0160S$
 $\text{C} = 1,3440 - 0,0025T$ $2\text{C} = 2,7121 - 0,0053T$
 $0,1401S = -9,2444$ $0,1479T = +7,9767$

§. 168. Nunc igitur habebimus

$S = -66,6980$ $L-S = 1,824113$
 $T = +53,9330$ $L+T = 1,731855$
 $\text{C} = +2,4497$ $L\text{C} = 0,389113$
 $\text{C} = +1,3092$ $L\text{C} = 0,082498$
 $s' = -0,75204$ $L-s' = 9,876241$
 $s' = +0,62626$ $L+s' = 9,796755$
 $v' = -0,6942$ $L-v' = 9,841484$
 $v' = +0,0244$ $L+v' = 8,387390$
 $y' = -0,1216$ $L-y' = 9,084933$
 $z' = +0,0200$ $L+z' = 9,301030$

R =

$\text{Q} = +7,0311$ $L\text{Q} = 0,847029$
 $\text{R} = +2,2561$ $L\text{R} = 0,353358$
 $\text{Q} = +5,7659$ $L\text{Q} = 0,760867$
 $\text{R} = +1,3339$ $L\text{R} = 0,125123$
 $V = -63,8498$ $L-V = 1,805160$
 $X = +1,7451$ $L+X = 0,241820$
 $Y = -13,6222$ $L-Y = 1,134241$
 $Z = +0,5356$ $L+Z = 9,728840$

§. 169. His iam valoribus inventis pro distantia

lunae $x = \frac{(1-kk') \sin \alpha}{1-k \cos \alpha}$ erit valoris ipsius n portio ab his

terminis pendens:

$n = \text{Præc.} - 0,3796 \text{ ek} \cos(r-s)$ $L\text{og. coeff.}$
 $+ 0,3069 \text{ ek} \cos(r+s)$ $9,579297$
 $- 0,3634 \text{ ek} \cos(2y-r+s)$ $9,560344$
 $+ 0,0099 \text{ ek} \cos(2y+r-s)$ $7,997004$
 $- 0,0775 \text{ ek} \cos(2y-r-s)$ $8,889425$
 $+ 0,0030 \text{ ek} \cos(2y+r+s)$ $7,484024$

et pro longitudine lunae

$d\theta = \text{Præc.} + 0,7520 \text{ ek} \cos(r-s)$
 $d\varphi = \text{Præc.} + 0,6263 \text{ ek} \cos(r+s)$
 $+ 0,6942 \text{ ek} \cos(2y-r+s)$
 $+ 0,0244 \text{ ek} \cos(2y+r-s)$
 $+ 0,1216 \text{ ek} \cos(2y-r-s)$
 $- 0,0200 \text{ ek} \cos(2y+r+s)$ T^2
cuius

160S

253T

1767

1 =

culus integrale n ponatur:

$$\phi = \text{Præc.} + \mathcal{E}' e^k \ln(r-s) + \mathfrak{R} e^k \ln(2\eta-r+s) + \mathfrak{U} e^k \ln(2\eta-r-s) + \mathcal{E} e^k \ln(r+s) + \mathfrak{X} e^k \ln(2\eta+r-s) + \mathfrak{S} e^k \ln(2\eta+r+s)$$

$$\begin{aligned} + 0,7520 &= (-\frac{1}{n}) \mathcal{E}' e^{-k} (\omega^k r^k) - \mathfrak{D} e^{-k} (\frac{1}{n} \mathfrak{U} e^{-k} \ln(r-s) - \mathfrak{R} e^{-k} \ln(2\eta-r+s) - \mathfrak{S} e^{-k} \ln(2\eta+r-s)) \\ - 0,6263 &= (1 + \frac{1}{n}) \mathcal{E}' e^{-k} (\omega^k r^k) - \mathfrak{D} e^{-k} (\frac{1}{n} \mathfrak{U} e^{-k} \ln(r-s) - \mathfrak{R} e^{-k} \ln(2\eta-r+s) - \mathfrak{S} e^{-k} \ln(2\eta+r-s)) \\ + 0,6942 &= (2a-1 + \frac{1}{n}) \mathfrak{X}' + \mathfrak{X}' (\frac{2}{n} - r) + \mathfrak{D} (\frac{2}{n} - r) - \mathfrak{R} (r-1) \\ - 0,0244 &= (2a+1 - \frac{1}{n}) \mathfrak{X}' + \mathfrak{X}' (\frac{2}{n} - r) + \mathcal{E}' (\frac{2}{n} - r) - \mathfrak{D} (r + \frac{1}{n}) \\ + 0,1216 &= (2a-1 - \frac{1}{n}) \mathfrak{U}' + \mathfrak{U}' (\frac{2}{n} - r) + \mathfrak{D} (\frac{2}{n} - r) - \mathfrak{D} (r + \frac{1}{n}) \\ - 0,0200 &= (2a+1 + \frac{1}{n}) \mathfrak{S}' + \mathfrak{S}' (\frac{2}{n} - r) + \mathcal{E}' (\frac{2}{n} - r) - \mathfrak{R} (r - \frac{1}{n}) \end{aligned}$$

§. 170. Hinc aurem reperitur

$\mathcal{E}' = + 0,7467$	$\mathfrak{D}' = 9,873165$
$\mathcal{E}' = - 0,6185$	$\mathfrak{D}' = 9,791317$
$\mathfrak{U}' = + 0,8143$	$\mathfrak{R}' = 9,910800$
$\mathfrak{X}' = - 0,0142$	$\mathfrak{X}' = 8,150690$
$\mathfrak{U}' = + 0,2396$	$\mathfrak{U}' = 9,379550$
$\mathfrak{S}' = - 0,0061$	$\mathfrak{S}' = 7,788910$

§. 171.

§. 171. Quatenus ergo longitudo Lunae ab eccentricitate orbis pendet, erit

$\phi = \text{Præc.} + 0,201385 e \ln r$	$9,304026$
$- 0,021889 e \ln(2\eta-s)$	$8,340237$
$- 0,016368 e \ln(2\eta+s)$	$8,214002$
$+ 0,06615 ee \ln 2s$	$8,820508$
$+ 0,02332 ee \ln(2\eta-2s)$	$8,367825$
$+ 0,00840 ee \ln(2\eta+2s)$	$7,924429$
$+ 0,7476 e k \ln(r-s)$	$9,873165$
$- 0,6185 e k \ln(r+s)$	$9,791317$
$+ 0,8143 e k \ln(2\eta-r+s)$	$9,910800$
$- 0,0142 e k \ln(2\eta+r-s)$	$8,150690$
$+ 0,2396 e k \ln(2\eta-r-s)$	$9,379550$
$- 0,0061 e k \ln(2\eta+r+s)$	$7,788910$

log. coeff.

T 3

§. 172.

§. 172. Hae autem singulae inaequalitates ad numerum minorum secundorum reducendae dabunt :

		log. coeff.
$\phi =$ Praec.	$+ 701\frac{1}{2}, 1 \text{ fin } s$	2,845780
—	75, 8 fin (2 η - s)	1,879971
—	56, 7 fin (2 η + s)	1,753736
+	3, 8 fin 2 s	0,585551
+	1, 4 fin (2 η - 2 s)	0,132862
+	0, 5 fin (2 η + 2 s)	9,689472
+	140, 9 fin (r - s)	2,148800
—	116, 7 fin (r + s)	2,067000
+	153, 7 fin (2 η - r + s)	2,186530
—	2, 7 fin (2 η + r - s)	0,426300
+	45, 2 fin (2 η - r - s)	1,655200
+	1, 2 fin (2 η + r + s)	0,064600

Hic scilicet et inaequalitates, quas in capite praecedente invenimus, et istas in hoc capite eripias simul sum complexus, ut conjunctionem confectui exponeretur.

CAPUT

CAPUT XI.

INVESTIGATIO INAEQUALITATUM LUNAE
A PARALLAXI SOLIS PENDENTIUM.

§. 173.

Jam in formulis nostris primariis ad eos quoque terminos progrediamur, qui littera v sunt affecti, et quoniam est $1 : v$ ut distantia Solis media ad distantiam Lunae mediam a Terra, erit $1 : v$ ut parallaxis Lunae media ad parallaxim solis : ex quo inaequalitates Lunae, quae hinc oriuntur, a parallaxi solis pendere dicuntur. Quoniam vero valor ipsius v est valde parvus, quippe $\frac{2}{50}$ propemodum, alios terminos non contemplabimur, nisi qui per v ac per $v k$ et $v e$ sunt multiplicati, propterea quod magis compositi fiunt minimi.

§. 174. Ex terminis ergo iam inventis hic retineamus eos, qui sunt alicuius momenti, et cum iis nos determinandos coniungamus ; sic ergo :

$$\begin{aligned}
 \sqrt{Rd} = & \mathcal{A} \text{ cof } 2\eta + \mathcal{B} k \text{ of } r + \mathcal{C} k \text{ cf } (2\eta - r) + \mathcal{D} e \text{ cf } s + \mathcal{E} \text{ cf } (2\eta - s) \\
 & + \mathcal{F} k \text{ cf } (2\eta + s) + \mathcal{G} v \text{ cof } v + \mathcal{H} v k \text{ cof } (v - r) + \mathcal{I} v e \text{ cof } (v - s) \\
 & + \mathcal{J} v \text{ cof } 3\eta + \mathcal{K} v k \text{ cof } (v + r) + \mathcal{L} v e \text{ cof } (v + s) \\
 \phi = & \mathcal{A} \text{ cof } 2\eta \cdot \cdot + \mathcal{D} k \text{ cof } (2\eta - r) + \mathcal{P} e \text{ cof } s + \mathcal{Q} e \text{ cof } (2\eta - s) \\
 & + \mathcal{E} k \text{ cof } (2\eta + s) + \mathcal{R} s \text{ cof } (2\eta + s) \\
 & + \mathcal{F} v \text{ cof } v + \mathcal{H} v k \text{ cof } (v - r) + \mathcal{K} v e \text{ cof } (v - s) \\
 & + \mathcal{G} v \text{ cof } 3\eta + \mathcal{J} v k \text{ cof } (v + r) + \mathcal{L} v e \text{ cof } (v + s)
 \end{aligned}$$

Non

JT