De Numeris Amicabilibus.

Definitio.

§. I.

Bini Numeri vocantur amicabiles, si ita sint comparati, ut summa partium aliquotarum unius aequalis sit alteri numero, & vicissim summa partium aliquotarum alterius priori numero aequetur.

Sic isti numeri 220 & 284 sunt amicabiles; prioris enim 220 partes aliquotae junctim sumtae: 1+2+4+5+10+11+20+22+44+55+110 faciunt 284: & hujus numeri 284 partes aliquotae: 1+2+4+71+142 producunt priorem numerum 220.

Scholion.

§. II. Stifelius, qui primus hujusmodi numerorum mentionem fecit, casu hos duos numeros 220 & 284 contemplatus ad hanc speculationem deductus videtur; analysin enim ineptam existimat, cujus ope plura istiusmodi numerorum paria inveniantur. Cartesius vero analysin ad hoc negotium accommodare est conatus, regulamque tradidit, qua tria talium numerorum paria eruit, neque praeter ea Schotenius, qui multum in hac investigatione desudasse videtur, plura eruere valuit. Post haec tempora nemo fere Geometrarum ad hanc quaestionem magis evolvendam operam impendisse reperitur. Cum autem nullum sit dubium quin analysis quoque ex hac parte incrementa non contemnenda sit consecutura, si methodus aperiatur, qua multo plura hujusmodi numerorum paria investigare liceat, haud abs re fore arbitror, si methodos quasdam huc spectantes, in quas forte incidi, communicavero. In hunc finem autem sequentia praemittere necesse est.

Hy-

Hypothesis.

§. III. Si n denotet numerum quemcunque integrum positivum, cujusmodi numeri hic semper sunt intelligendi, omnium ejus divisorum summam hoc signo $\int n$ indicabo, ita ut character $\int$ numero cuipiam præfixus summam omnium ejusdem numeri divisorum denotet; sic erit $\int 6 = 1 + 2 + 3 + 6 = 12$.

Corollarium. 1.

§. IV. Quoniam inter divisores cujusvis numeri hic ipse numerus refertur, partes aliquotæ autem censentur divisores ipso numero excepto, manifestum est summam partium aliquotarum numeri n exprimi per $\int n - n$.

Coroll. 2.

§. V. Quoniam numerus primus nullos alios divisores admittit præter unitatem & se ipsum, si n sit numerus primus erit $\int n = 1 + n$. Cum autem casu $n = 1$ sit $\int 1 = 1$, patet unitatem non recte numeris primis annumerari.

Lemma. 1.

§. VI. Si m & n fuerint numeri inter se primi, ut præter unitatem nullum habeant divisorem communem, tum erit $\int mn = \int m . \int n$, seu summa divisorum producti mn æqualis est producto ex summis divisorum utriusque numeri m & n.

Productum enim mn primo habet singulos divisores utriusque factoris m & n, tum vero insuper divisibile est per producta ex singulis divisoribus numeri m in singulos divisores numeri n. Hi vero omnes ipsius mn divisores junctim prodeunt, si $\int m$ per $\int n$ multiplicetur.

Coroll. 1.

§. VII. Si numerorum m & n uterque sit primus, ideoque $\int m = 1 + m$ & $\int n = 1 + n$, erit summa divisorum producti $\int mn =$

(1+m)

20-JAN-2005 21:25 ScreenScan

$(1+m)(1+n) = 1+m+n+mn$. Si præterea p sit numerus primus diversus ab m & n, erit $\int mnp = \int mn.\int p = \int m.\int n.\int p = (1+m)(1+n)(1+p)$. Hincque summa divisorum cujusque numeri, qui est productum ex quotcunque numeris primis diversis, facile assignabitur.

Coroll. 2.

§. VIII. Si m, n, & p non quidem sint numeri primi, sed tamen ejusmodi, ut præter unitatem nullum habeant divisorem communem, tum mn & p erunt numeri inter se primi, ac propterea $\int mnp = \int mn.\int p$. Cum autem sit $\int mn = \int m.\int n$: erit $\int mnp = \int m.\int n.\int p$.

Scholion.

§. IX. Nisi factores $m.n.p$ sint numeri inter se primi, summa divisorum producti, prout per lemma indicatur, non est justa. Cum enim secundum lemma singuli divisores factorum $m.n.p$ inter divisores producti mnp referantur, si haberent divisorem communem, is inter divisores producti bis numeraretur; at dum quæstio de summa divisorum cujuspiam numeri instituitur, nullum divisorem bis numerare oportet. Hinc si m & n sint numeri primi ac $m = n$, non erit $\int nn = \int n.\int n = (1+n)^2 = 1+2n+nn$, sed habebitur $\int m = 1+n+nn$, neque divisorem n bis poni convenit. Cum igitur per hoc lemma summa divisorum cujusque numeri, qui est productum ex quotcunque numeris primis diversis, recte assignetur, residuum est, ut pro factoribus æqualibus regula tradatur, cujus ope summa divisorum producti definiri queat.

Lemma. 2.

§. X. Si n sit numerus primus, erit $\int n^2 = 1+n+n^2$, $\int n^3 = 1+n+n^2+n^3$; $\int n^4 = 1+n+n^2+n^3+n^4$, & generatim erit

$$\int n^k = 1+n+n^2+\cdots+n^k = \frac{n^{k+1}-1}{n-1}.$$

Co-

20-JAN-2005 21:26 ScreenScan

Coroll. 1.

§. XI. Cum sit $\int n = 1 + n$, erit $\int n^2 = \int n + n^2$, vel etiam $\int n^2 = 1 + n \int n$. Simili modo erit $\int n^3 = \int n^2 + n^3$, vel etiam $\int n^3 = 1 + n \int n^2$; porroque $\int n^4 = \int n^3 + n^4$ seu $\int n^4 = 1 + \int n^3$ & ita porro. Sicque ex cognita summa divisorum cujusque potestatis n^k facile summa divisorum potestatis sequentis n^{k+1} assignatur, cum sit $\int n^{k+1} = \int n^k + n^{k+1}$ seu $\int n^{k+1} = 1 + n \int n^k$.

Coroll. 2.

§. XII. Quo summae divisorum facilius per factores exprimi queant, notandum est esse $\int n^3 = (1+n)(1+n^2) = (1+n^2)\int n$; $\int n^5 = (1+n^2+n^4)\int n$; $\int n^7 = (1+n^2+n^4+n^6)\int n = (1+n^2)(1+n^4)\int n$: sicque summae divisorum potestatum imparium semper per factores exhiberi possunt: at potestatum parium summae divisorum quandoque erunt numeri primi.

Coroll. 3.

§. XIII. Hinc igitur facile tabula condi poterit, qua non solum numerorum primorum, sed etiam potestatum ipsorum summae divisorum exhibeantur. Cujusmodi Tabulam hic adjicere visum est, in qua omnium numerorum primorum millenario non majorum, eorumque potestatum ad tertiam usque & altiores pro minoribus numeris summae divisorum per factores expressae traduntur.

Num-

20-JAN-2005 21:26 ScreenScan

Num.	Summa Divisorum.
2	3
2^{2}	7
2^{3}	3.5
2^{4}	31
2^{5}	$3^{2}.7$
2^{6}	127
2^{7}	3.5.17.
2^{8}	7.73
2^{9}	3.11.31
2^{10}	23.89
2^{11}	$3^{2}.5.7.13$
2^{12}	8191
2^{13}	3.43.127
2^{14}	7.31.151
2^{15}	3.5.17.257
2^{16}	131071
2^{17}	$3^{3}.7.19.73$
2^{18}	524287
2^{19}	$3.5^{2}.11.31.41$
2^{20}	$7^{2}.127.337$
2^{21}	3.23.89.683
2^{22}	47.178481
2^{23}	$3^{2}.5.7.13.17.241$
2^{24}	31.601.1801
2^{25}	3.2731.8191
2^{26}	7.73.262657
2^{27}	3.5.29.43.113.127
2^{28}	233.1103.2089
2^{29}	$3^{2}.7.11.31.151.331$
2^{30}	2147483647
2^{31}	3.5.17.257.65537
2^{32}	7.23.89.599479
2^{33}	3.43691.131071
2^{34}	31.71.127.122921
2^{35}	$3^{3}.5.7.13.19.37.73.109$
2^{36}	223.616318177.

Numeri	Summ. Divisorum.
3	2^{2}
3^{2}	13
3^{3}	$2^{3}.5$
3^{4}	11^{2}
3^{5}	$2^{2}.7.13$
3^{6}	1093
3^{7}	$2^{4}.5.41$
3^{8}	13.757
3^{9}	$2^{2}.11^{2}.61$
3^{10}	23.3851
3^{11}	$2^{3}.5.7.13.73$
3^{12}	797161
3^{13}	$2^{2}.547.1093$
3^{14}	$11^{2}.13.4561$
3^{15}	$2^{5}.5.17.41.193$
5	2.3
5^{2}	31
5^{3}	$2^{2}.3.13$
5^{4}	11.71
5^{5}	$2.3^{2}.7.31$
5^{6}	19531.
5^{7}	$2^{2}.3.13.313$
5^{8}	19.31.829
5^{9}	2.3.11.71.521
7	2^{3}
7^{2}	3.19
7^{3}	$2^{4}.5^{2}$
7^{4}	2801
7^{5}	$2^{3}.3.19.43$
7^{6}	29.4733
7^{7}	$2^{5}.5^{2}.1201$
7^{8}	$3^{2}.19.37.1063$
7^{9}	$2^{3}.11.191.2801$
7^{10}	329 54457.

Numeri	Summa Divisorum.
11	$2^{2}.3$
11^{2}	7.19
11^{3}	$2^{3}.3.61$
11^{4}	5.3221
11^{5}	$2^{2}.3^{2}.7.19.37$
11^{6}	43.45319
11^{7}	$2^{4}.3.61.7321$
11^{8}	7.19.1772893
11^{9}	$2^{2}.3.5.3221.13421$
13	2.7
13^{2}	3.61
13^{3}	$2^{2}.5.7.17$
13^{4}	30941
13^{5}	2.3.7.61.157
13^{6}	5229043
13^{7}	$2^{2}.5.7.17.14281$
17	2.3^{2}
17^{2}	307
17^{3}	$2^{2}.3^{2}.5.29$
17^{4}	88741
17^{5}	$2.3^{3}.7.13.307$
19	$2^{2}.5$
19^{2}	3.127
19^{3}	$2^{3}.5.181$
19^{4}	151.911
19^{5}	$2^{2}.3.5.7^{2}.127$
23	$2^{3}.3$
23^{2}	7.79.
23^{3}	$2^{4}.3.5.53$
23^{4}	292561

20-JAN-2005 21:27 ScreenScan

Num	Sum Divisorum	Num.	Sum. Divisorum	Num	Summa Divisor.	Num.	Sum Divisorum.
29	$2.3.5$	67	$2^2.17$	109	$2.5.11.$	163	$2^2.41$
29^2	13.67	67^2	$3.7^2.31$	109^2	$3.7.571$	163^2	$3.7.19.67$
29^3	$2^2.3.5.421$	67^3	$2^3.5.17.449$	109^3	$2^2.5.11.13.457$	163^3	$2^3.5.41.2657$
31	2^5	71	$2^3.3^2$	113	$2.3.19$	167	$2^3.3.7$
31^2	3.331	71^2	5113	113^2	13.991	167^2	28057
31^3	$2^6.13.37$	71^3	$2^4.3^2.2521$	113^3	$2^2.3.5.19.1277$	167^3	$2^4.3.5.7.2789$
37	2.19	73	2.37	127	2^7	173	$2.3.29$
37^2	$3.7.67$	73^2	3.1801	127^2	3.5419	173^2	67.449
37^3	$2^2.5.2603$	73^3	$2^2.5.13.37.41$	127^3	$2^8.5.1613$	173^3	$2^2.3.5.29.41.73$
41	$2.3.7$	83	$2^2.3.7$	131	$2^2.3.11$	179	$2^2.3^2.5$
41^2	1723	83^2	19.367	131^2	17293	179^2	7.4603
41^3	$2^2.3.7.29^2$	83^3	$2^3.3.5.7.13.53.$	131^3	$2^3.3.11.8581.$	179^3	$2^3.3^2.5.37.433$
43	$2^2.11$	89	$2.3^2.5$	137	$2.3.23$	181	$2.7.13$
43^2	3.631	89^2	8011	137^2	$7.37.73$	181^2	$3.79.139$
43^3	$2^3.5^2.11.37$	89^3	$2^2.3^2.5.17.233.$	137^3	$2^2.3.5.23.1877.$	181^3	$2^2.7.13.16381$
47	$2^4.3$	97	2.7^2	139	$2^2.5.7$	191	$2^6.3$
47^2	37.61	97^2	3.3169	139^2	$3.13.499$	191^2	$7.13^2.31$
47^3	$2^5.3.5.13.17$	97^3	$2^2.5.7^2.941$	139^3	$2^3.5.7.9661$	191^3	$2^7.3.17.29.37$
53	2.3^3	101	$2.3.17$	149	$2.3.5^2$	193	2.97
53^2	7.409	101^2	10303	149^2	$7.31.103$	193^2	$3.7.1783$
53^3	$2^2.3^3.5.281$	101^3	$2^2.3.17.5101.$	149^3	$2^2.3.5^2.11101.$	193^3	$2^2.5^2.97.149$
59	$2^2.3.5$	103	$2^3.13$	151	$2^3.19$	197	$2.3^2.11$
59^2	3541	103^2	3.3571	151^2	$3.7.1093$	197^2	19.2053
59^3	$2^3.3.5.1741$	103^3	$2^4.5.13.1061$	151^3	$2^4.13.19.877.$	197^3	$2^2.3^2.5.11.3881$
61	2.31	107	$2^2.3^3$	157	2.79	199	$2^3.5$
61^2	$3.13.97$	107^2	$7.13.127$	157^2	3.8269	199^2	3.13267
61^3	$2^2.31.1861$	107^3	$2^3.3^3.5^2.229$	157^3	$2^2.5^2.17.29.79$	199^3	$2^4.5^2.19801$

Num.

20-JAN-2005 21:27 ScreenScan

Num.	Summa Divisor.	Num.	Summa Divisorum	Num.	Summa Divisorum	Num.	Summa. Divisorum
211	$2^2.53$	263	$2^3.3.11$	313	2.157	373	$2.11.17$
211^2	$3.13.31.37$	263^2	$7^2.13.109$	313^2	3.181^2	373^2	$3.7^2.13.73$
211^3	$2^3.53.113.197$	263^3	$2^4.3.5.11.6917.$	313^3	$2^2.5.97.101.157.$	373^3	$2^2.5.11.17.13913$
223	$2^5.7$	269	$2.3^3.5$	317	$2.3.53.$	379	$2^2.5.19$
223^2	3.16651	269^2	$13.37.151$	317^2	7.14401	379^2	$3.61.787$
223^3	$2^6.5.7.4973.$	269^3	$2^2.3^3.5.97.373.$	317^3	$2^2.3.5.13.53.773$	379^3	$2^3.5.19.71821$
227	$2^2.3.19$	271	$2^4.17$	331	$2^2.83$	383	$2^7.3$
227^2	73.709	271^2	3.24571	331^2	$3.7.5233$	383^2	147073
227^3	$2^3.3.5.19.5153.$	271^3	$2^5.17.36721$	331^3	$2^3.29.83.1889.$	383^3	$2^8.3.5.14669$
229	$2.5.23$	277	2.139	337	2.13^2	389	$2.3.5.13$
229^2	$3.97.181$	277^2	$3.7.19.193$	337^2	$3.43.883$	389^2	7.21673
229^3	$2^2.5.13.23.2017$	277^3	$2^2.5.139.7673$	337^3	$2^2.5.13^2.41.277.$	389^3	$2^2.3.5.13.29.2609$
233	$2.3^2.13$	281	$2.3.47$	347	$2^2.3.29$	397	2.199
233^2	7.7789	281^2	109.727	347^2	$7.13.1327$	397^2	$3.31.1699$
233^3	$2^2.3^2.5.13.61.89$	281^3	$2^2.3.13.47.3037$	347^3	$2^3.3.5.29.12041$	397^3	$2^2.5.199.15761$
239	$2^4.3.5$	283	$2^2.71$	349	$2.5^2.7$	401	$2.3.67$
239^2	19.3019	283^2	$3.73.367$	349^2	$3.19.2143$	401^2	7.23029
239^3	$2^5.3.5.13^4$	283^3	$2^3.5.71.8009$	349^3	$2^2.5^2.7.60901$	401^3	$2^2.3.37.41.53.67$
241	2.11^2	293	$2.3.7^2$	353	$2.3.59$	409	$2.5.41$
241^2	3.19441	293^2	$86143.$	353^2	19.6577	409^2	3.55897
241^3	$2^2.11^2.113.257$	293^3	$2^2.3.5^2.7^2.17.101$	353^3	$2^2.3.5.17.59.733$	409^3	$2^2.5.41.83641$
251	$2^2.3^2.7$	307	$2^2.7.11$	359	$2^3.3^2.5$	419	$2^2.3.5.7$
251^2	43.1471	307^2	$3.43.733$	359^2	$7.37.499$	419^2	13.13537
251^3	$2^3.3^2.7.17^2.109$	307^3	$2^3.5^3.7.11.13.29$	359^3	$2^4.3^2.5.13.4957$	419^3	$2^3.3.5.7.41.2141$
257	$2.3.43$	311	$2^3.3.13$	367	$2^4.23$	421	2.211
257^2	61.1087	311^2	19.5107	367^2	$3.13.3463$	421^2	3.59221
257^3	$2^2.3.5^2.43.1321$	311^3	$2^4.3.13.137.353$	367^3	$2^5.5.23.13469$	421^3	$2^2.13.17.211.401$

20-JAN-2005 21:27 ScreenScan

Num.	Summa Divisorum	Num.	Summa Divisor.	Num.	Summa Divisorum	Num.	Summa Divisorum
431	$2^4.3^3$	479	$2^5.3.5$	547	$2^2.137$	601	2.7.43
431^2	7.67.397	479^2	43.5347	547^2	3.163.613	601^2	3.13.9277
431^3	$2^6.3^3.293.317$	479^3	$2^6.3.5.89.1289$.	547^3	$2^3.5.137.29921$.	601^3	$2^2.7.43.313.577$
433	2.7.31	487	$2^3.61$	557	$2.3^2.31$	607	$2^5.19$
433^2	3.37.1693	487^2	3.7.11317	557^2	$7^2.6343$	607^2	3.13.9463
433^3	$2^2.5.7.31.18749$	487^3	$2^4.5.37.61.641$	557^3	$2^2.3^2.5^2.17.31.73$.	607^3	$2^6.5^2.19.7369$
439	$2^3.5.11$	491	$2^2.3.41$	563	$2^2.3.47$	613	2.307
439^2	$3.31^2.67$	491^2	37.6529	563^2	31.10243	613^2	3.125461
439^3	$2^4.5.11.173.557$.	491^3	$2^3.3.41.149.809$	563^3	$2^3.3.5.29.47.1093$	613^3	$2^2.5.53.307.709$
443	$2^2.3.37$	499	$2^2.5^3$	569	2.3.5.19	617	2.3.103
443^2	7.28099	499^2	$3.7.109^2$	569^2	$7^2.6619$	617^2	97.3931.
443^3	$2^3.3.5^4.37.157$	499^3	$2^3.5^3.13.61.157$	569^3	$2^2.3.5.19.161881$	617^3	$2^2.3.5.103.38069$
449	$2.3^2.5^2$	503	$2^3.3^2.7$	571	$2^2.11.13$	619	$2^2.5.31$
449^2	97.2083	503^2	13.19501	571^2	3.7.103.151	619^2	3.19.6733
449^3	$2^2.3^2.5^2.100801$.	503^3	$2^4.3^2.5.7.25301$.	571^3	$2^3.11.13.163021$.	619^3	$2^3.5.13.31.14737$
457	2.229	509	2.3.5.17	577	2.17^2	631	$2^3.79$
457^2	3.7.9967	509^2	43.6037	577^2	3.19.5851	631^2	3.307.433
457^3	$2^2.5^2.229.4177$.	509^3	$2^2.3.5.17.281.461$	577^3	$2^2.5.13^2.17^2.197$.	631^3	$2^4.79.199081$
461	2.3.7.11	521	$2.3^2.29$	587	$2^2.3.7^2$	641	2.3.107
461^2	373.571	521^2	$31^2.283$	587^2	547.631	641^2	7.58789
461^3	$2^2.3.7.11.106261$	521^3	$2^2.3^2.29.135721$.	587^3	$2^3.3.5.7^2.34457$.	641^3	$2^2.3.107.205441$
463	$2^4.29$	523	$2^2.131$	593	$2.3^3.11$	643	$2^2.7.23$
463^2	3.19.3769	523^2	3.13.7027	593^2	163.2161	643^2	3.97.1423
463^3	$2^5.5.13.17.29.97$	523^3	$2^3.5.17.131.1609$	593^3	$2^2.3^3.5^2.11.13.541$	643^3	$2^3.5^2.7.23.8269$
467	$2^2.3^2.13$	541	2.271	599	$2^3.3.5^2$	647	$2^3.3^4$
467^2	19.11503	541^2	3.7.13963	599^2	7.51343	647^2	211.1987
467^3	$2^3.3^2.5.13.113.193$	541^3	$2^2.13.271.11257$	599^3	$2^4.3.5^2.17.61.173$	647^3	$2^4.3^4.5.41.1021$

N[um.]
65[3]
65[3]^2
65[3]^3
65[9]
65[9]^2
65[9]^3
66[1]
66[1]^2
66[1]^3
673
673^2
673^3
677
677^2
677^3
683
683^2
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691
691^2
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701
701^2
701^3
709
709^2
709^3

Num.

20-JAN-2005 21:28 ScreenScan

Num.	Summa Divisorum	Num.	Summa Divisorum	Num.	Summa Divisor.	Num.	Summa Divisorum.
653	$2.3.109$	719	$2^4.3^2.5$	773	$2.3^2.43$	839	$2^3.3.5.7$
653^2	$7.13^2.19^2$	719^2	487.1063	773^2	598303	839^2	704761
653^3	$2^2.3.5.109.42641$	719^3	$2^5.3^2.5.53.4877$	773^3	$2^2.3^2.5.43.59753$	839^3	$2^4.3.5.7.109.3229$
659	$2^2.3.5.11$	727	$2^3.7.13$	787	$2^2.197$	853	$2.7.61$
659^2	13.33457	727^2	3.176419	787^2	$3.37^2.151$	853^2	$3.43.5647$
659^3	$2^3.3.5.11.17.53.241$	727^3	$2^4.5.7.13.17.3109$	787^3	$2^3.5.197.241.257$	853^3	$2^2.5.7.13.29.61.193$
661	2.331	733	2.367	797	$2.3.7.19$	857	$2.3.11.13$
661^2	3.145861	733^2	$3.19.9439$	797^2	157.4051	857^2	735307
661^3	$2^2.331.218461$	733^3	$2^2.5.13.367.4133$	797^3	$2^2.3.5.7.19.63521$	857^3	$2^2.3.5^2.11.13.37.397$
673	2.337	739	$2^2.5.37$	809	$2.3^4.5$	859	$2^2.5.43$
673^2	3.151201	739^2	$3.7.26041$	809^2	$7.13.19.379$	859^2	3.246247
673^3	$2^2.5.337.45293$	739^3	$2^3.5.37.273061$	809^3	$2^2.3^4.5.229.1429$	659^3	$2^3.5.43.137.2693$
677	$2.3.113$	743	$2^3.3.31$	811	$2^2.7.29$	863	$2^5.3^3$
677^2	459007	743^2	552793	811^2	$3.31.73.97$	863^2	$7^2.15217$
677^3	$2^2.3.5.113.45833$	743^3	$2^4.3.5^2.31.61.181$	811^3	$2^3.7.13.29.41.617$	863^3	$2^6.3^3.5.13.17.337$
683	$2^2.3^2.19$	751	$2^4.47$	821	$2.3.137$	877	2.439
683^2	7.66739	751^2	$3.7.26893$	821^2	$7.229.421$	877^2	$3.7.37.991$
683^3	$2^3.3^2.5.19.46649$	751^3	$2^5.47.282001$	821^3	$2^2.3.137.337021$	877^3	$2^2.5.439.76913$
691	$2^2.173$	757	2.379	823	$2^3.103$	881	$2.3^2.7^2$
691^2	$3.19.8389$	757^2	$3.13.14713$	823^2	$3.7.43.751$	881^2	19.40897
691^3	$2^3.173.$[illegible]	757^3	$2^2.5^2.73.157.379$	823^3	$2^4.5.103.67733$	881^3	$2^2.3^2.7^2.388081$
701	$2.3^3.13$	761	$2.3.127$	827	$2^2.3^2.23$	883	$2^2.13.17$
701^2	492103	761^2	579883	827^2	684757	883^2	3.260191
701^3	$2^2.3^3.13.17.97.149$	761^3	$2^2.3.17.127.17033$	827^3	$2^3.3^2.5.13.23.5261$	883^3	$2^3.5.13.17.77969$
709	$2.5.71$	769	$2.5.7.11$	829	$2.5.83$	887	$2^3.3.37$
709^2	$3.7.23971$	769^2	$3.31.6367$	829^2	$3.211.1087$	887^2	13.60589
709^3	$2^2.5.71.$[illegible]	769^3	$2^2.5.7.11.$[illegible]	829^3	$2^2.5.17^2.29.41.83$	887^3	$2^4.3.5.29.37.2713$

Num.	Summa Divisorum	Num.	Summa Divisorum.
907	$2^2.227$	971	$2^2.3^5$
907^2	3.7.39217	971^2	13.79.919
907^3	$2^3.5^2.227.16453.$	971^3	$2^3.3^5.197.2393$
911	$2^4.3.19$	977	2.3.163
911^2	830833	977^2	7.136501
911^3	$2^5.3.19.29.41.349$	977^3	$2^2.3.5.53.163.1801$
919	$2^3.5.23$	983	$2^3.3.41$
919^2	3.7.13.19.163	983^2	103.9391
919^3	$2^4.5.23.37.101.113$	983^3	$2^4.3.5.13.41.7433$
929	2.3.5.31	991	$2^5.31$
929^2	157.5503	991^2	$3.7.13^2.277$
929^3	$2^2.3.5.31.431521.$	991^3	$2^6.31.491041$
937	2.7.67	997	2.499
937^2	3.292969	997^2	3.13.31.823
937^3	$2^2.5.7.67.87797$	997^3	$2^2.5.499.99401$
941	2.3.157		
941^2	811.1093		
941^3	$2^2.3.13.157.34057$		
947	$2^2.3.79$		
947^2	7.277.463		
947^3	$2^3.3.5.79.89681$		
953	$2.3^2.53$		
953^2	181.5023		
953^3	$2^2.3^2.5.53.90821$		
967	$2^3.11^2$		
967^2	3.67.4657		
967^3	$2^4.5.11^2.13.7193$		

Schg-

20-JAN-2005 21:28 ScreenScan

Scholion.

§. XIV. Usus hujus tabulæ est amplissimus in quæstionibus circa divisores & partes aliquotas versantibus resolvendis. Ejus enim ope cuiusque numeri propositi summa divisorum facili negotio inveniri potest, qua reperta, si inde ipse numerus propositus auferatur, remanebit ejus summa partium aliquotarum. Ex quo statim constat, hujus tabulæ subsidio numeros amicabiles, quos sum traditurus, facile explorari posse, utrum sint justi nec ne? Quemadmodum autem ope hujus tabulæ cujusvis numeri summa divisorum cognosci possit, in sequenti lemmate explicabo.

Lemma. 3.

§. XV. Proposito quocunque numero ejus summa divisorum sequenti modo colligitur.

Cum omnis numerus sit vel primus vel productum ex primis, resolvatur numerus propositus in suos factores primos, & qui inter se fuerint æquales, conjunctim exprimantur. Hoc modo numerus propositus semper ad hujusmodi formam redigetur $m^{\alpha} \cdot n^{\beta} \cdot p^{\gamma} \cdot q^{\delta}$ &c. existentibus m, n, p, q, &c. numeris primis. Posito ergo numero proposito $= N$ cum sit $N = m^{\alpha} \cdot n^{\beta} \cdot p^{\gamma} \cdot q^{\delta}$ &c. & factores $m^{\alpha}, n^{\beta}, p^{\gamma}, q^{\delta}$ &c. inter se primi; erit $\int N = \int m^{\alpha} \cdot \int n^{\beta} \cdot \int p^{\gamma} \cdot \int q^{\delta}$. &c. & valores $\int m^{\alpha}, \int n^{\beta}, \int p^{\gamma}, \int q^{\delta}$ &c. ex tabula adjuncta patebunt.

1. Exempl. *Sit numerus propositus* $N = 360$.

Resoluto hoc numero in suos factores primos erit $N = 2^3 \cdot 3^2 \cdot 5$, ideoque $\int 360 = \int 2^3 \int 3^2 \cdot \int 5 = 3 \cdot 5 \cdot 13 \cdot 2 \cdot 3$, ob $\int 2^3 = 3 \cdot 5$; $\int 3^2 = 13$; $\int 5 = 2 \cdot 3$.

Unde his factoribus ordinatis fiet $\int 360 = 2 \cdot 3^2 \cdot 5 \cdot 13 = 1170$.

20-JAN-2005 21:29 ScreenScan

2. Exempl. *Explorentur numeri 2620 & 2924 utrum sint amicabiles nec ne?*

Cum sit $2620 = 2^2 . 5 . 131$ & $2924 = 2^2 . 17 . 43$, examen ita instituetur.

Numeri propositi	2620	2924
per factores expressi	$2^2 . 5 . 131$	$2^2 . 17 . 43$
summæ divisorum	$7 . 6 . 132$	$7 . 18 . 44$
sive	5544	5544
Summæ partium aliquotarum	2924	2620

Cum igitur summæ partium aliquotarum sint numeris reciproce æquales, patet propositos numeros esse amicabiles.

Scholion.

§. XVI. His igitur præmissis, quæ ad inventionem divisorum cujusque numeri pertinent, ipsum problema de investigatione numerorum amicabilium aggrediar, atque scrutabor, quemadmodum hujusmodi numeros ratione summæ divisorum inter se comparatos esse oporteat, quo deinceps facilius eorum inventio per regulas post tradendas suscipi queat.

Problema generale.

§. XVII. *Invenire numeros amicabiles, hoc est duos numeros hujus indolis, ut alter æqualis sit summæ partium aliquotarum alterius.*

Solutio.

Sint m & n duo hujusmodi numeri amicabiles, & per hypothesin $\int m$ & $\int n$ summæ divisorum eorumdem. Erit numeri m summa partium aliquotarum $= \int m - m$, & numeri n summa partium aliquotarum $= \int n - n$. Hinc ex natura numerorum amicabilium nascentur hæ duæ æquationes:

$\int m$

20-JAN-2005 21:29 ScreenScan

$$\int m - m = n \ \& \ \int n - n = m$$
$$\text{five} \quad \int m = \int n = m + n.$$

Numeri ergo amicabiles m & n primo habere debent eandem summam divisorum, tum vero oportet, ut hæc communis divisorum summa æqualis sit aggregato ipsorum numerorum $m+n$.

Coroll. 1.

§. XVIII. Problema ergo huc reducitur, ut quærantur duo ejusmodi numeri, qui habeant candem divisorum summam, hæcque æqualis sit aggregato ipsorum numerorum.

Coroll. 2.

§. XIX. Ipsa quidem problematis ratio exigit, ut bini numeri quæsiti sint inter se inæquales: sin autem desiderentur æquales, ut sit $m = n$, fiet $\int n = 2n$ & $\int n - n = n$: hujus scilicet numeri geminati n summa partium aliquotarum ipsi fiet æqualis, quæ est proprietas numeri perfecti. Ergo quilibet numerus perfectus repetitus numeros exhibet amicabiles.

Coroll. 3.

§. XX. Sin autem numeri amicabiles m & n, ut natura quæstionis postulat, sint inæquales, manifestum est, alterum esse redundantem alterum deficientem; summa scilicet partium aliquotarum alterius ipso erit major, alterius vero ipso minor.

Scholion.

§. XXI. Ex hac quidem generali proprietate parum adjumenti consequimur ad numeros amicabiles inveniendos, eo quod ista analyseos species, cujus ope æquationem $\int m = \int n = m+n$ evolvere liceat, etiamnunc penitus sit inculta. Ob quem defectum formulas magis particulares contemplari cogimur, ex quarum indole regulas speciales pro inventione numerorum amicabilium derivare

rivare liceat; quorsum etiam pertinet regula Cartesiana a Schotenio commemorata. Ac primo quidem, etiamsi non constet, utrum dentur numeri amicabiles inter se primi nec ne? formulas generales ita restringam, ut numeri amicabiles factorem communem obtineant.

Problema Particulare.

§. XXII. *Invenire indolem numerorum amicabilium, qui communem habeant factorem.*

Solutio.

Sit a communis factor numerorum amicabilium, quorum alter ponatur $= am$, alter $= an$; sint vero tam m & a, quam n & a numeri inter se primi, ut utriusque divisorum summa per præcepta data reperiri queat. Cum igitur primo utriusque eadem esse debeat divisorum summa, fiet $\int a.\int m = \int a.\int n$, ideoque $\int m = \int n$. Deinde vero necesse est ut sit $\int a.\int m$ seu $\int a.\int n$ ipsorum numerorum æqualis aggregato $am + an$, unde habetur $\frac{a}{\int a} = \frac{\int m}{m+n} = \frac{\int n}{m+n}$.
Positis ergo numeris amicabilibus am & an, primo esse oportet $\int m = \int n$, tum vero requiritur ut sit $a(m+n) = \int a.\int m$.

Coroll. 1.

§. XXIII. Si ergo pro m & n ejusmodi numeri jam fuerint eruti ut sit $\int m = \int n$: tum numerus a investigari debet, ut sit $\frac{a}{\int a} = \frac{\int m}{m+n}$, seu ex ratione, quam numerus ad summam divisorum suorum tenere debet, ipse numerus a erit investigandus.

Coroll. 2.

§. XXIV. Si factor communis a fuerit datus, quæstio ad inventionem numerorum m & n reducitur, qui prouti vel primi

vel

20-JAN-2005 21:30 ScreenScan

vel compositi ex duobus pluribusve primis assumuntur, quoniam tum divisorum summae actu exhiberi possunt, regulae speciales ad eos inveniendos tradi poterunt.

Coroll. 3.

§. XXV. Statim autem perspicitur utrumque numerum m & n primum esse non posse: quare casus simplicissimus extat, si alter primus, alter vero productum ex duobus numeris primis assumetur. Tum uterque productum ex duobus, pluribusve numeris primis statui poterit, unde innumerae regulae speciales pro inveniendis numeris amicabilibus derivari poterunt.

Scholion.

§. XXVI. Diversae ergo numerorum amicabilium formae, quae hinc nascuntur, sequenti modo repraesentari poterunt. Sit a utriusque communis factor, & p, q, r, s &c. numeri primi, quorum nullus sit divisor communis factoris a: atque numerorum amicabilium formae erunt:

Forma Prima - - - $\begin{cases} apq \\ ar \end{cases}$

Forma Secunda - - - $\begin{cases} apq \\ ars \end{cases}$

Forma Tertia - - - $\begin{cases} apqr \\ as \end{cases}$

Forma Quarta - - - $\begin{cases} apqr \\ ast \end{cases}$

Forma Quinta - - - $\begin{cases} apqr \\ astu \end{cases}$

&c.

Quanquam numerus harum formarum in infinitum augeri potest,

20-JAN-2005 21:30 ScreenScan

rest, minime tamen hinc concludere licet, in his formis omnes numeros amicabiles contineri. Primum enim, dum hic litteræ p, q, r, s, t, &c. numeros primos diversos significant, non verisimile est, nullos dari numeros amicabiles, in quibus non occurrant potestates ejusdem numeri primi. Deinde pariter non constat, utrum non dentur numeri amicabiles, qui vel nullum habeant factorem communem a, vel in quibus factor hic non prorsus sit idem: veluti si darentur numeri amicabiles hujus formæ $m^{\alpha}P$ & $m^{\epsilon}Q$, in quibus exponentes α & ϵ essent diversi; quæ forma propterea in superioribus non contineretur, etiamsi P & Q essent producta ex meris numeris primis inter se diversis. Ex his perspicitur quæstionem de numeris amicabilibus latissime patere; eamque ob hoc ipsum tam esse difficilem, ut solutio completa vix sit expectanda. Solutionibus igitur particularibus equidem tantum incumbam, & varias methodos aperiam, quarum ope ex formulis traditis plures numeros amicabiles mihi elicere licuit. Quælibet autem forma duplicem mihi suspeditavit methodum, prout factor communis a vel datus assumitur, vel ipse quæritur; hasque methodos in sequentibus problematibus exponam.

Problema. 1.

§. XXVII. Invenire numeros amicabiles primæ formæ apq & ar, si factor communis a sit datus.

Solutio.

Cum p, q, & r sint numeri primi, atque $\int r = \int p \cdot \int q$ seu $r+1 = (p+1)(q+1)$, ponatur $p+1 = x$ & $q+1 = y$, fietque $r = xy - 1$, ideoque x & y ejusmodi esse oportet numeros, ut tam $x-1$, & $y-1$ quam $xy-1$ sint numeri primi. Deinde ut $a(x-1)(y-1)$ & $a(xy-1)$ sint numeri amicabiles, oportet ut eorum aggregatum $a(2xy-x-y)$ æquale sit summæ diviso-

rum

20-JAN-2005 21:31 ScreenScan

rum alterutrius $xy\int a$: unde nanciscimur hanc aequationem $xy\int a = 2axy - ax - ay$ seu $y = \frac{ax}{(2a-\int a)x - a}$. Sit brevitatis gratia $\frac{a}{2a-\int a} = \frac{b}{c}$, & $\frac{b}{c}$ sit valor fractionis $\frac{a}{2a-\int a}$ ad minimos terminos reductae, eritque $y = \frac{bx}{cx-b}$ seu $cy = \frac{bcx}{cx-b} = b + \frac{bb}{cx-b}$, unde habebimus $(cx-b)(cy-b) = bb$. Cum igitur $cx-b$ & $cy-b$ sint factores ipsius bb, quadratum cognitum bb in ejusmodi binos factores resolvi debet, quorum uterque numero b auctus fiat per c divisibilis, & quoti x & y inde emergentes ita sint comparati, ut $x-1$, $y-1$, & $xy-1$ evadant numeri primi. Quae conditio quoties obtineri poterit, quod quidem pro quovis valore ipsius a assumto statim dispicitur, toties obtinebuntur numeri amicabiles, qui erunt $a(x-1)(y-1)$ & $a(xy-1)$ Q. E. I.

Coroll.

§. XXVIII. Porro igitur pro a alii aliique numeri accipiantur, unde valores b & c innotescant, regulae emergent particulares, quarum ope numeri amicabiles, si qui in eo genere dentur, facile eruentur.

Regula I.

§. XXIX. Sit factor communis a potestas quaecunque binarii, puta $a = 2^n$ erit $\int a = 2^{n+1} - 1$, ideoque $2a - \int a = 1$, unde erit $\frac{a}{2a-\int a} = 2^n$, & propterea $b = 2^n$ & $c = 1$. Hinc oritur $(x-2^n)(y-2^n) = 2^{2n}$.

Quare

20-JAN-2005 21:31 ScreenScan

Quare cum 2^{2n} alios non habeat factores nisi potestates binarii, erit:

$$x-2^n = 2^{n+k} \qquad x = 2^{n+k}+2^n$$
$$y-2^n = 2^{n-k} \text{ seu } y = 2^{n-k}+2^n$$

Quocirca dispiciendum est, an ejusmodi valor pro k detur, ut sequentes tres numeri

$$x-1 = 2^{n+k}+2^n-1$$
$$y-1 = 2^{n-k}+2^n-1$$
$$xy-1 = 2^{2n+1}+2^{2n+k}+2^{2n-k}-1$$

sint numeri primi. Quod si succedat erunt numeri amicabiles:

$$2^n(2^{n+k}+2^n-1)(2^{n-k}+2^n-1)$$
$$2^n(2^{2n+1}+2^{2n+k}+2^{2n-k}-1)$$

Vel sit $n-k=m$ seu $n=m+k$, fietque

$$x-1 = 2^m(2^{2k}+2^k)-1 = q$$
$$y-1 = 2^m(1+2^k)-1 = p$$
$$xy-1 = 2^{2m}(2^{2k+1}+2^{3k}+2^k)-1 = r$$

qui numeri, quoties fuerint primi, praebebunt numeros amicabiles.

Casus 1.

§. XXX. Sit $k=1$; & numeri amicabiles obtinebuntur, quoties sequentes tres numeri fuerint primi:

3.2

20-JAN-2005 21:31 ScreenScan

$3.2^m - 1$; $6.2^m - 1$; & $18.2^{2m} - 1$

Tum enim positis:

$$p = 3.2^m - 1;\quad q = 6.2^m - 1;\quad \&\quad r = 18.2^{2m} - 1$$

numeri amicabiles erunt: $2^{m+1}pq$ & $2^{m+1}r$, ob $n = m + k = m + 1$. Haecque est regula Cartesii a Schotenio tradita.

Exemplum. 1.

§. XXXI. Sit $m = 1$; eritque

$p = 3.2 - 1 = 5$ numerus primus.
$q = 6.2 - 1 = 11$ numerus primus.
$r = 18.4 - 1 = 71$ numerus primus.

hinc ergo oriuntur numeri amicabiles;

$$2^2.5.11 \quad \& \quad 2^2.71$$

Sive 220 & 284, qui sunt minimi omnium, qui exhiberi possunt.

Exempl. 2.

§. XXXII. Sit $m = 2$, eritque $2^m = 4$ & $2^{2m} = 16$ atque

$p = 3.4 - 1 = 11$ numerus primus.
$q = 6.4 - 1 = 23$ numerus primus.
$r = 18.16 - 1 = 287$ numerus non-primus.

hincque adeo nulli numeri amicabiles oriuntur.

Exempl. 3.

§. XXXIII. Sit $m = 3$, eritque $2^m = 8$ & $2^{2m} = 64$ atque

$p = 3.8 - 1 = 23$ primus
$q = 6.8 - 1 = 47$ primus
$r = 18.64 - 1 = 1151$ primus.

20-JAN-2005 21:32 ScreenScan

Ergo hinc numeri amicabiles erunt:

$2^4 . 23 . 47$ & $2^4 . 1151$
five 17296 & 18416.

Exempla feqq.

§. 34. Hæc exempla cum fequentibus, in quibus exponenti m majores valores tribuuntur, commodius uno confpectu ita repræfentari poterunt.

Sit $m =$	1	2	3	4	5	6	7	8
erit $p =$	5	11	23	47	95*	191	383	767*
$q =$	11	23	47	95*	191	383	767*	1535*
$r =$	71	287*	1151	4607*	18431*	73727	294911	1179647

Ubi numeri non-primi afterifcis funt notati: unde hinc tantum terni numeri amicabiles obtinentur, nempe

$$\text{I.}\begin{cases} 2^2 . 5 . 11 \\ 2^2 . 71 \end{cases} \quad \text{II.}\begin{cases} 2^4 . 23 . 47 \\ 2^4 . 1151 \end{cases} \quad \text{III.}\begin{cases} 2^7 . 191 . 383 \\ 2^7 . 73727 \end{cases}$$

Ulterius autem progredi non licet, quoniam valores ipfius r nimis funt magni, quam ut dignofci poffit, utrum fint primi nec ne? Tabulæ namque numerorum primorum adhuc conftructæ vix ultra 100000 porriguntur.

Cafus. II.

§. XXXV. Sit $k = 2$ & valores litterarum $p, q, r,$ qui debent effe primi, erunt:

$$p = 5 . 2^m - 1$$

$$q = 20 . 2^m - 1$$

$$r = 100 . 2^{2m} - 1$$

quo-

20-JAN-2005 21:32 ScreenScan

quorum cum postremus semper sit per ternarium divisibilis, ob $2^{2m} = 3\alpha + 1$ & $r = 300\alpha + 99$, hinc nulli numeri amicabiles consequuntur.

Casus. III.

§. XXXVI. Ponatur $k = 3$, eritque

$$p = 9 \cdot 2^{m} - 1$$

$$q = 72 \cdot 2^{m} - 1$$

$$r = 648 \cdot 2^{2m} - 1$$

quorum cum nullus necessario videatur divisorem admittere, valores ipsorum p, q, r, ex valoribus simplicioribus ipsius m oriundos hic conjunctim repraesentabo.

$m =$	1	2	3	4	5
$p =$	17	35*	71	143*	287*
$q =$	143*	287*	575*	1151	2303*
$r =$	2591	10367*	41471*	165887	663551

Hinc ergo, quoniam ulterius progredi non licet, nulli numeri amicabiles inveniuntur.

Casus. IV.

§. XXXVII. Ponatur $k = 4$, & sequentes tres numeri debebunt esse primi.

$$p = 17 \cdot 2^{m} - 1$$

$$q = 272 \cdot 2^{m} - 1$$

$$r = 4624 \cdot 2^{2m} - 1$$

20-JAN-2005 21:32 ScreenScan

Ubi cum r ſemper ſit multiplum ternarii, patet hinc nullos prodire numeros amicabiles.

Caſus V.

§. XXXVIII. Ponatur $k = 5$, & ſequentes tres numeri debebunt eſſe primi.

$$p = 33 \cdot 2^m - 1$$

$$q = 1056 \cdot 2^m - 1$$

$$r = 34848 \cdot 2^{2m} - 1$$

Ubi ſtatim patet caſum $m = 1$ eſſe inutilem, cum det $p = 65$. Sit ergo $m = 2$, fietque

$p = 131$, $q = 4223^*$, $r = 557567$

ubi cum q non ſit primus, & majores valores pro m ob defectum tabularum numerorum primorum examini ſubjici nequeant, neque hinc etiam novi numeri amicabiles eruuntur. At vero ob eandem rationem majores valores ipſi k tribuere non licet.

Scholion.

§. XXXIX. Quoniam poteſtates binarii pro a poſitæ valorem ipſius c in fractione $\frac{b}{c} = \frac{a}{2a - fa}$ unitati æqualem reddiderunt, hincque ſolutiones obtinere licuit, alios valores pro a, qui pariter ipſi c valorem $= 1$ inducant, ponam. Inter hos autem imprimis ſunt notandi, qui ex hac forma $a = 2^n(2^{n+1} + e)$ naſcuntur, ſi quidem $2^{n+1} + e$ ſit numerus primus, tum enim ſit $2a -$

fa

fa $= e+1$, & $\frac{b}{c} = \frac{2^n(2^{n+1}+e)}{e+1}$: si igitur $e+1$ sit divisor numeratoris $2^n(2^{n+1}+e)$ valor ipsius c fiet itidem $=1$.

Regula. II.

§. XL. Sit factor communis $a = 2^n(2^{n+1}+2^k-1)$, at $2^{n+1}+2^k-1$ numerus primus, erit ob $e+1=2^k$, fractio

$$\frac{c}{b} = \frac{2^n(2^{n+1}+2^k-1)}{2^k} = 2^{n-k}(2^{n+1}+2^k-1),$$

si quidem non sit $k>n$. Hac ergo hypothesi habebimus $b=2^{n-k}(2^{n+1}+2^k-1)$ & $c=1$. Quadratum ergo bb in duos ejusmodi factores $(x-b)(y-b)$ resolvendum est, ex quibus non solum valores numerorum $x-1=p$ & $y-1=q$, sed etiam $xy-1=r$ fiant numeri primi. Cujusmodi casus si eruere liceat, erunt numeri amicabiles apq & ar. Verum hic notandum est eos casus rejiciendos esse, in quibus aliquis numerorum primorum p, q, r prodit divisor ipsius a, seu aequalis $2^{n+1}+2^k-1$, quia a per nullum alium numerum primum est divisibile.

Sit $n-k=m$, seu $n=m+k$, erit $a=2^{m+k}(2^{m+k+1}+2^k-1)$ & $b=2^m(2^{m+k+1}+2^k-1.)$ Jam quia $2^{m+k+1}+2^k-1$ debet esse numerus primus, ponatur $2^{m+k+1}+2^k-1=f$ seu $f=$

$2^k(2^{m+1}+1)-1$, ut sit $a = 2^{m+k}f$ & $b = 2^m f$; erit $bb = 2^{2m}ff = (x-b)(y-b)$. Nunc ob f numerum primum, numerus $2^{2m}ff$ duplici modo in genere in duos factores resolvetur. Priori modo fiet $(x-b)(y-b) = 2^{m-\alpha}f \cdot 2^{m+\alpha}f$, ideoque

$$x = 2^{m-\alpha}f + 2^m f; \qquad p = (2^{m-\alpha} + 2^m)f - 1$$

$$y = 2^{m+\alpha}f + 2^m f; \qquad q = (2^{m+\alpha} + 2^m)f - 1$$

$$\& \; r = (2^{2m+1} + 2^{2m+\alpha} + 2^{2m-\alpha})ff - 1$$

qui tres numeri p, q, r debent esse primi. Posteriori modo resolutio fiet ita:

$$(x-b)(y-b) = 2^{m\pm\alpha} \cdot 2^{m\mp\alpha}ff, \text{ unde fit}$$

$$x = 2^{m\pm\alpha} + 2^m f; \qquad p = 2^{m\pm\alpha} + 2^m f - 1$$

$$y = 2^{m\mp\alpha}ff + 2^m f; \qquad q = (2^{m\mp\alpha}f + 2^m)f - 1$$

$$\& \; r = (2^{2m+1}f + 2^{2m\pm\alpha} + 2^{2m\mp\alpha}ff)f - 1$$

& quoties p, q, r hoc modo prodeunt numeri primi, inde oriuntur numeri amicabiles apq & ar.

Casus. 1.

§. XLI. Sit $k = 1$, erit $a = 2^{m+1}(2^{m+2}+1)$, $b = $ $2m$

$2^m(2^{m+2}+1)$ atque $f = 2^{m+2}+1$, qui numerus debet esse primus. Cum ergo sit $(x-b)(y-b) = 2^{2m}ff$, erit

vel	vel
$p = (2^{m-n}+2^m)f-1$	$p = 2^{m\pm n}+2^m f-1$
$q = (2^{m+n}+2^m)f-1$	$q = (2^{m+n}f+2^m)f-1$
$r = (2^{2m+1}+2^{2m+n}+2^{2m-n})ff-1$	$r = (2^{2m+1}f+2^{2m\pm n}+2^{2m+n}ff)f-1$

Notandum autem est, ut $2^{m+2}+1$ sit numerus primus, exponentem $m+2$ esse oportere potestatem binarii: valores ergo ipsius m erunt: 0, 2, 6, 14, &c. At casus $m = 0$ rejici debet, ob nullum valorem ipsius n assignabilem.

Exemplum. 1.

§. XLII. Sit ergo $m = 2$, ut sit $a = 8.17$ & $b = 4.17 = 68$ atque $f = 17$. Cum igitur esse debeat $(x-b)(y-b) = 4^4.17$, erit resolutione in factores instituenda:

$x-68=$	2	4	8	34
$y-68=$	8.17*	1156	578	136
$x=$	70	72	76	102
$y=$	2380	1224	646	204
$p=$	69*	71	75*	101
$q=$	2379*	1223	645*	203*
$r=$	166599*	88127*	49095*	20807

Hinc ergo nulli numeri amicabiles obtinentur.

Exem-

20-JAN-2005 21:34 ScreenScan

Exemplum. 2.

§. XLIII. Sit $m = 6$, ut $a = 2^7 . 257$; $b = 2^6 . 257$ & $f = 257$. Cum igitur sit

$$(x - b)(y - b) = 2^{12} . 257^2$$

Resolutio ita institui debebit:

$x - 16448 =$	$32 . 257$
$y - 16448 =$	$128 . 257$
$x =$	24672
$y =$	40344
$p =$	24671
$q =$	49343^*
$r =$	$- - - -$

Valores ex reliquis factoribus oriundi adhuc magis fiunt magni quam ut, an primi sint nec ne, judicari possit.

Casus reliqui.

§. XLIV. Cum $f = 2^{m+k+1} + 2^k - 1$ debeat esse numerus primus, quaeramus primo casus simpliciores, quibus hoc evenit, cum casus nimis compositos evolvere non liceat. Sit ergo $k = 2$, & ob $f = 2^{m+3} + 3$, valores idonei pro m erunt: 1, 3, 4: sit $k = 3$, erit $f = 2^{m+4} + 7$, & valores idonei pro m erunt 2, 4, 8. Casu $k = 4$, est $f = 2^{m+5} + 15$, & m erit 1, vel 3 neque ulterius progredi licet.

Exempl. 1.

§. XLV. Ponamus ergo $k = 2$, & $m = 1$, erit $f = 19$ & $a = 8 . 19$ atque $b = 2 . 19 = 38$, unde fiet

(x-

20-JAN-2005 21:34 ScreenScan

$$(x-38)(y-38) = 2^2 . 19^2 = 1444$$

& resolutio dabit.

$x-38=$	2	4	
$y-38=$	722	361	Neuter scilicet factor
$x=$	40	..	assumi potest impar
$y=$	760	imp:	
$p=$	39*		

Quia hic jam p non est primus, patet hinc nullos numeros amicabiles resultare.

Exempl. 2.

§. XLVI. Ponamus $k = 2$ & $m = 3$, ut sit $f = 67$ erit $a = 32.67$ & $b = 8.67 = 536$: und sit

$$(x-536)(y-536) = 2^6 . 67^2$$

$x-536$	268	16	
$y-536$	1072	17956	reliqui valores pro p præbent
$x=$	804	552	numeros per 3 divisibiles,
$y=$	1608	...	quos propterea omisi. Se-
$p=$	803*	1551*	quentia exempla ad nimis
$q=$	1607	...	magnos numeros deducunt.

Regula III.

§. XLVII. Sit ut ante $a = 2^n(2^{n+1}+2^k-1)$ & $2^{n+1}+2^k-1 = f$ numerus primus, et in fractione $\frac{b}{c} = \frac{2^n(2^{n+1}+2^k-1)}{2^k}$ sit $k > n$; eritque $b = 2^{n+1}+2^k-1$ & $c = 2^{k-n}$. Ponamus $k-n=m$, ut sit $k = m+n$, erit $a = 2^n(2^{n+1}+2^{m+n}-1)$; $b =$

20-JAN-2005 21:35 ScreenScan

$b = 2^{n+1} + 2^{m+n} - 1 = f$ & $c = 2$; unde hæc habebitur æquatio

$$(2^m x - b)(2^m y - b) = bb$$

Cum autem $b = f$ sit numerus primus, alia resolutio locum non invenit præter $1 \cdot bb$: ex qua fit

$$x = \frac{1+b}{2^m} \quad \& \quad y = \frac{b(1+b)}{2^m} \quad \text{sive}$$

$$x = 2^n + 2^{n+1-m} \quad \& \quad y = (2^{n+1} + 2^{m+n} - 1)(2^n + 2^{n+1-m})$$

Jam notandum est hos quatuor numeros esse oportere primos:

$$f = 2^{n+1} + 2^{m+n} - 1$$

$$p = x - 1; \quad q = y - 1; \quad \& \quad r = xy - 1$$

atque necesse est ut sit $m < n+1$. Quibus conditionibus si satisfiat, erunt numeri amicabiles: apq & ar.

Casus 1.

§. XLVIII. Sit $m = 1$, erit $f = 2^{n+2} - 1$; $x = 2^{n+1}$, & $p = 2^{n+1} - 1$, fieri autem nequit, ut simul & f & p sit numerus primus, nisi casu $n = 1$, quo vero fit $q = 27$. Ergo ex hypothesi $m = 1$ nulli oriuntur numeri amicabiles.

Casus 2.

§. XLIX. Sit ergo $m = 2$, ut sit $f = 3 \cdot 2^{n+1} - 1$; $x = 3 \cdot 2^{n-1}$ & $y = 3 \cdot 2^{n-1}(3 \cdot 2^{n+1} - 1)$, atque $a = 2^n \cdot f$.

Sequen-

20-JAN-2005 21:35 ScreenScan

Sequentes ergo quatuor numeri debent esse primi:

$f = 3 \cdot 2^{n+1} - 1$; $p = 3 \cdot 2^{n-1} - 1$; $q = 3 \cdot 2^{n-1}(3 \cdot 2^{n+1} - 1) - 1$ & $r = 9 \cdot 2^{2n-2}(3 \cdot 2^{n+1} - 1) - 1$, unde formantur hæc exempla;

$n =$	1	2	3	4	5
$f =$	11	23	47	95*	191
$p =$	2	5	11	- -	47
$q =$	32*	137	563	- -	9167*
$r =$	98*	827	6767*	- -	
		valet			

hincque ergo ex $n = 2$, & $a = 4 \cdot 23$ nascuntur numeri amicabiles:

$$\begin{cases} 4 \cdot 23 \cdot 5 \cdot 137 \\ 4 \cdot 23 \cdot 827 \end{cases}$$

Casus ceteri.

§. L. Si $m = 3$, iterum vel f vel p fit divisibile per 3, quod idem evenit si $m = 5$, vel 7, &c. Sit ergo $m = 4$; erit $f = 9 \cdot 2^{n+1} - 1$; $x = 9 \cdot 2^{n-3}$ & $y = 9 \cdot 2^{n-3}(9 \cdot 2^{n+1} - 1)$ & $a = 2^n \cdot f$, unde formantur hæc exempla:

$n =$	1	4	5	6
$f =$	35*	287*	575*	1151
$x =$	*	* * *	* * *	72
$y =$	*	* * *	* * *	82872
$p =$	*	* * *	* * *	71
$q =$	*	* * *	* * *	82871*
$r =$	*	* * *	* * *	* * *

Ne-

20-JAN-2005 21:35 ScreenScan

Neque ergo hinc neque ex majoribus valoribus ipsi m tribuendis numeros amicabiles elicere licet.

Regula. IV.

§. LI. Possunt etiam aliae expressiones pro factore communi a inveniri, ex quibus fractionis $\frac{b}{c}$ denominator c vel unitati, vel potestati binarii fiat aequalis. Fingamus namque $a = 2^{n}(g-1)(h-1)$, ut sint $g-1$ & $h-1$ numeri primi; erit $\int a = (2^{n+1}-1)gh = 2^{n+1}gh - gh$; at est $2a = 2^{n+1}gh - 2^{n+1}g - 2^{n+1}h + 2^{n+1}$ unde fit

$$2a - \int a = gh - 2^{n+1}g - 2^{n+1}h + 2^{n+1}$$

Ponatur $2a - \int a = d$, erit $gh - 2^{n+1}(g+h) + 2^{n+1} = d$ & $(g - 2^{n+1})(h - 2^{n+1}) = d - 2^{n+1} + 2^{2n+2}$: unde per resolutionem in factores ejusmodi valores pro g & h elici debent, ut $g-1$ & $h-1$ fiant numeri primi, eritque tum $a = 2^{n}(g-1)(h-1)$ & $\frac{b}{c} = \frac{a}{d}$.

I. Ponamus $n = 1$, erit $(g-4)(h-4) = d + 12$, ubi ut $d + 12$ duos obtineat factores pares, sequentes prodibunt valores:

Sit $d = 4$; erit $(g-4)(h-4) = 16 = 2.8$, unde $g = 6$, $h = 12$; $a = 2.5.11$ atque $\frac{b}{c} = \frac{2.5.11}{4}$ ergo $b = 5.11$ & $c = 2$.

Sit