

The soccer ball and the solution of equations of the fifth degree

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Math 102: Topics in Geometry
Dartmouth College

Winter 2011
Time TBA

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1 Introduction

The quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ has been known in practice (though not expressed in this way) for about four thousand years. More complicated formulas for the solution of cubic and quartic equations by radicals were found in the 1500s. In 1824, Abel published his proof that the quintic equation can't be solved by radicals, but this was not the end of the line: In 1884, Klein showed how to solve the quintic using transcendental functions arising from the study of the icosahedron. This is possible because the icosahedron has symmetry group A_5 , which is the simple group standing in the way of an expression in terms of radicals.

In 1989 Curt McMullen and I showed how to solve the quintic using a tower of purely iterative algorithms. The key ingredient was a dynamical system with the symmetries of the icosahedron, or as we like to say, 'with the symmetries of the soccer ball'. The soccer ball has the same symmetries as the icosahedron: Combinatorially, it is a *truncated icosahedron*, with 12 pentagons corresponding to the vertices of the icosahedron and 20 hexagons

corresponding to the faces. But up until now, the soccer ball has played no direct role in the solution of the quintic.

In this course, our goal will be to compress the tower of iterations for solving the quintic by solving for all five roots at once. The key ingredient will be the ‘soccer map’, an iteration intimately tied to the soccer ball. This is joint, ongoing work with Scott Crass.

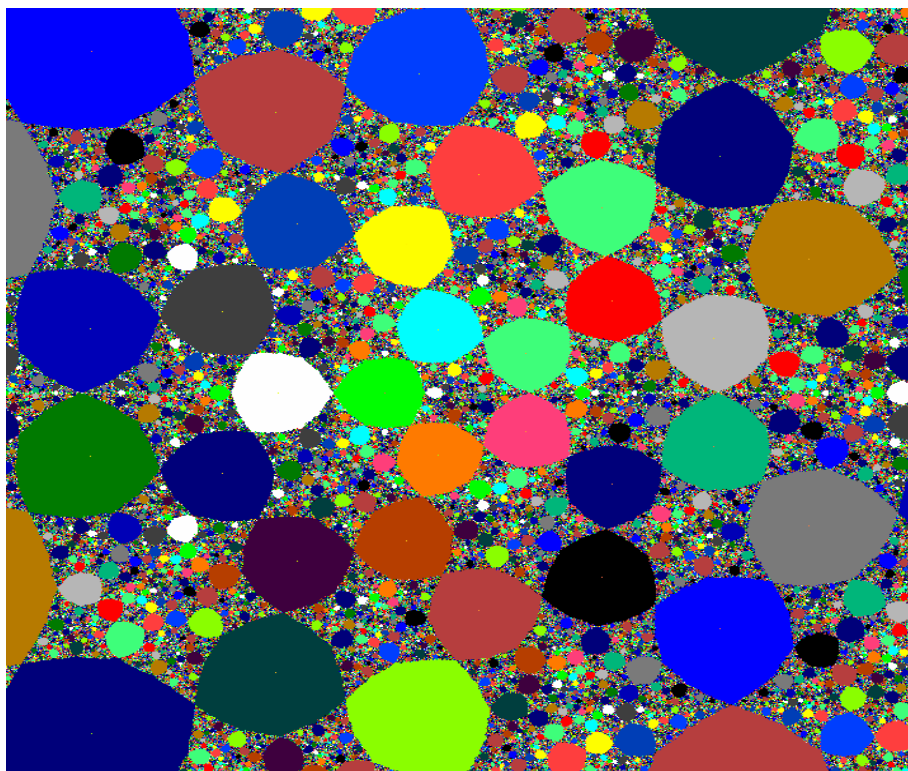


Figure 1: The basins of attraction of the soccer map. Is this the one true soccer ball?