

Deformations of Chaotic Billiards and a New ‘Wall Formula’ for Heating Rate

Alex Barnett

October 2000

Collaborators:
Doron Cohen
Rick Heller

Lyman Laboratory of Physics, Harvard University

Funding: National Science Foundation, ITAMP

Outline of today's talk

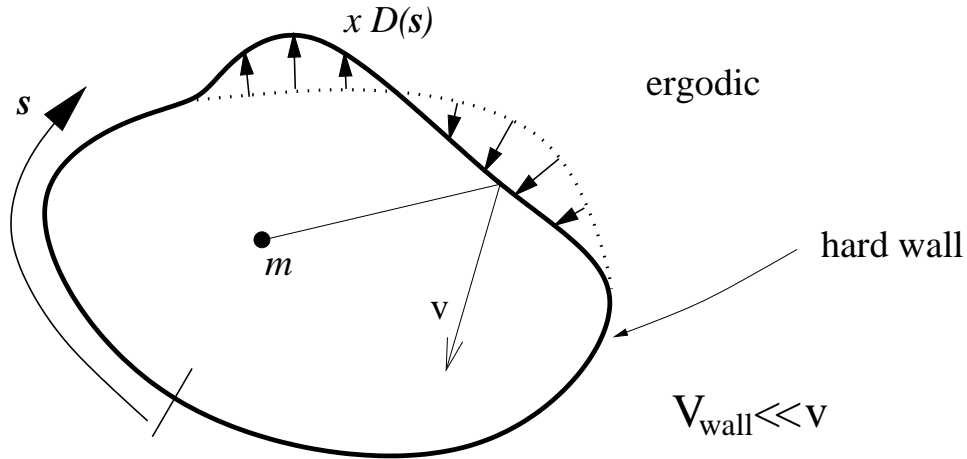
- Deforming billiards + motivation
- Key statements :
 1. Special class of deformations
 2. Vergini-Saraceno numerical method
 3. Improve 'wall formula'
- Theory of heating + 'wall formula' (classical)
- Explain 'special' deformations
- Quasi-orthogonality on the boundary (quantum)
- Improved 'wall formula' in action

SEE PAPERS: Alex Barnett, Doron Cohen and Eric J. Heller

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Deforming billiard (cavity) systems



$D(\mathbf{s}) =$ deformation shape function

$x(t) = A \sin \omega t$ periodic 'driving'

Question: At what rate is the 'gas' particle heated up?

Motivations

- Dissipation rate of vibrations of nuclei (3D)
 - never considered ω -dependence
- Driven mesoscopic 2D quantum dots (*e.g.* $x =$ gate voltage)
 - find heating rate of electrons

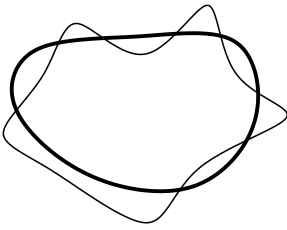
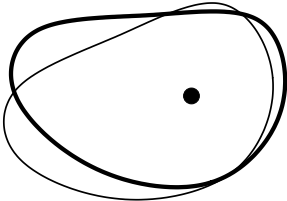
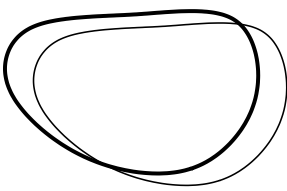
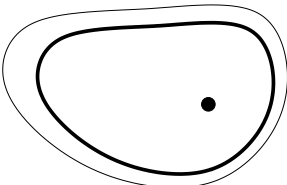
1) Special class of deformations

How does heating rate depend on deformation $D(\mathbf{s})$?

$$\text{heating } \frac{d}{dt} \langle \mathcal{H} \rangle = \mu(\omega) \cdot \frac{1}{2} (A\omega)^2$$

$\mu(\omega)$ = friction coefficient

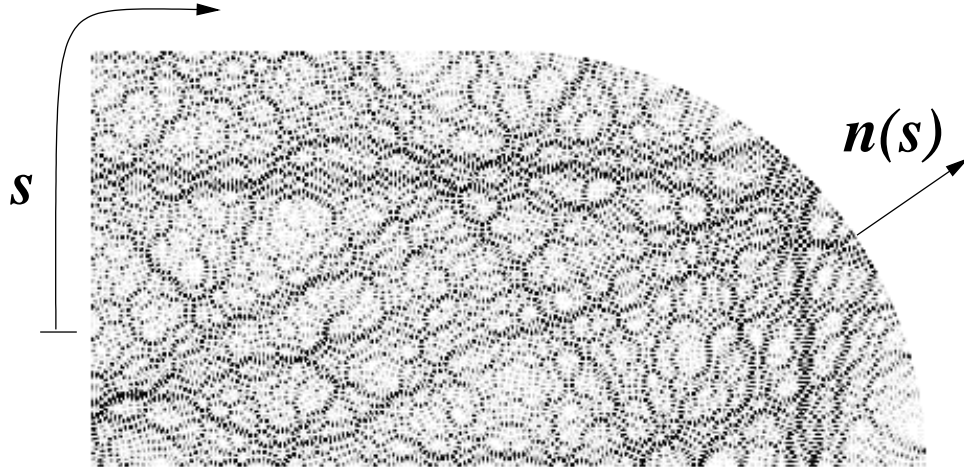
For low frequency $\omega \ll$ collision rate:

Generic		$\mu(\omega) \sim \text{const}$
Rotation		$\mu(\omega) \sim \omega^2$
Translation		$\mu(\omega) \sim \omega^4$
Dilation		$\mu(\omega) \sim \omega^4$

Does **not** depend on billiard shape or chaoticity

Special class surprise: **friction vanishes at dc** $\mu(\omega \rightarrow 0) = 0$

2) Vergini-Saraceno numerical method



eigenstate ψ_n

boundary function $\varphi_n \equiv \mathbf{n} \cdot \nabla \psi_n$

Quasi-orthogonality on boundary:

$$\oint (\mathbf{r} \cdot \mathbf{n}) ds \varphi_n(\mathbf{s}) \varphi_m(\mathbf{s}) \propto \delta_{nm} + \text{“error”} \left(\frac{E_n - E_m}{\hbar} \right)$$

V-S numerical method for finding eigenstates ψ_n

- 10^3 times more efficient than any other known method!
- finds clusters of eigenstates simultaneously
- needs “error” small close to diagonal

BUT No-one has known size of “error”!

I have shown: mean square “error”(ω) = $a \omega^4$

Due to $\mu(\omega) \sim \omega^4$ for **dilation** deformation

3) Improved ‘wall formula’ estimate for $\mu(0)$

Nuclear physics interest (last 25 years):

- seek analytic estimate of friction $\mu(0)$ given $D(\mathbf{s})$
- assumed uncorrelated collisions (strong chaos)
→ ‘wall formula’
- they knew $\mu(0) = 0$ for translations and rotations
→ *ad hoc* corrections

But now know special class of $D(\mathbf{s})$ for which $\mu(0) = 0$
(even for strong chaos)

We show: there is consistent way to *subtract* all special components of a general $D(\mathbf{s})$

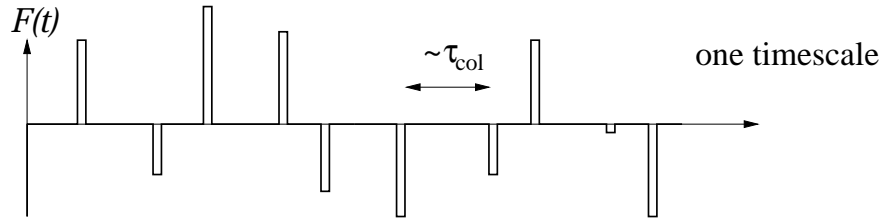
...**now** applying wall formula gives *improved* estimate of $\mu(0)$.

This replaces all *ad hoc* corrections

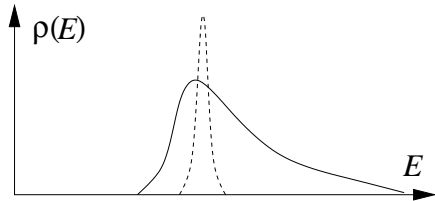
Theory of heating rate: energy spreading

Particle energy gets random ‘kicks’: $\dot{\mathcal{H}} = -\dot{x}(t)\mathcal{F}(t)$

where generalized ‘force’ on parameter $\mathcal{F}(t) \equiv -\frac{\partial \mathcal{H}}{\partial x}(t)$

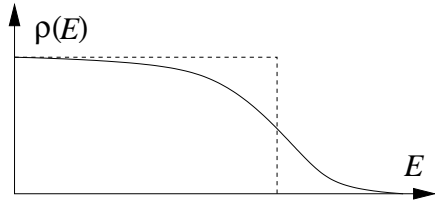


Energy diffusion rate $D_E \propto \tilde{C}_E(\omega) \equiv$ **power spectrum** of $\mathcal{F}(t)$



Causes irreversible energy growth
(Jarzynski, Cohen)

Why? D_E increases with E



Friction coefficient $\mu(\omega) \propto \tilde{C}_E(\omega)$
... relation depends on $\rho(E)$

The ‘wall formula’: white noise approximation (WNA)

Assume $\mathcal{F}(t) \approx$ white noise $\rightarrow \tilde{C}_E(\omega) = \text{const}$ (flat spectrum)

$$\tilde{C}_E(0) \xrightarrow{\text{ergodicity}} b_E \cdot \oint [D(\mathbf{s})]^2 ds, \quad \text{‘wall formula’ (Swiatecki)}$$

Some $D(\mathbf{s})$ obey WNA well, others badly...

Explanation of ‘special’ deformations

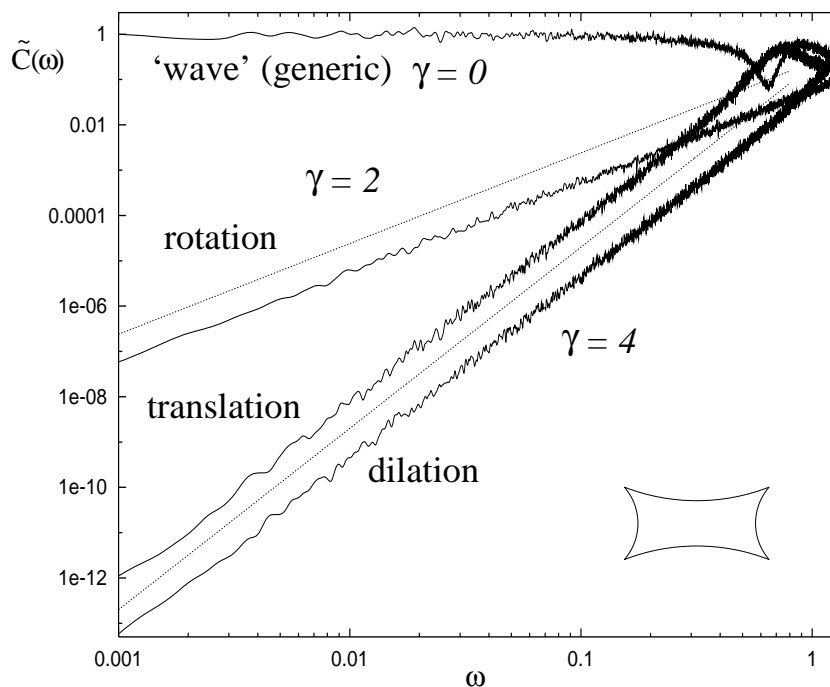
Some $D(\mathbf{s})$, WNA **fails**: $\tilde{C}_E(0) = 0$ (even in strong chaos)

Could always write $\mathcal{F}(t) = \left(\frac{d}{dt}\right)^n \mathcal{G}(t)$

\Rightarrow power spectra $\tilde{C}_E(\omega) = \omega^{2n} \tilde{C}_G(\omega)$ ($d/dt \xrightarrow{\text{FT}} i\omega$)

Special deformations: $\mathcal{G}(t) =$ some function of $(\mathbf{r}(t), \mathbf{p}(t))$

$\Rightarrow \tilde{C}_G(0)$ finite $\Rightarrow \tilde{C}_E(0)$ vanishes



Generic $\tilde{C}_G(\omega) \sim \omega^0 \rightarrow$ power laws $\tilde{C}_E(\omega) \sim \omega^\gamma$, $\gamma = 2n$

Improved estimate for $\tilde{C}_E(0)$ in action

- COMPONENTS OF GENERAL DEFORMATION:

Linear subspaces of $D(\mathbf{s})$: ‘special’ \perp WNA-good

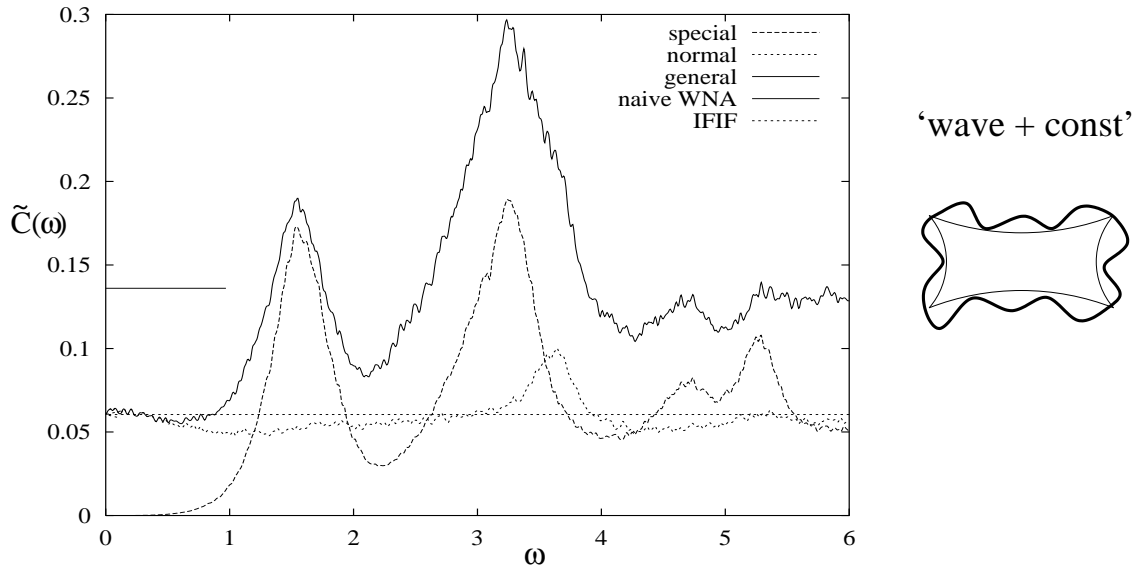
$$\text{Orthogonality : } 1 \perp 2 \Leftrightarrow \oint ds D_1(\mathbf{s}) D_2(\mathbf{s}) = 0$$

- SUBTRACT SPECIAL COMPONENT:

Make orthonormal set $\{D_i(\mathbf{s})\}$ of special defs, $i = 1 \cdots 1 + \frac{1}{2}d(d+1)$

$$D_{\perp}(\mathbf{s}) = D(\mathbf{s}) - \sum_i \alpha_i D_i(\mathbf{s}), \quad \text{components } \alpha_i = \oint ds D_i(\mathbf{s}) D(\mathbf{s})$$

- NOW APPLY WNA TO $D_{\perp}(\mathbf{s})$:



Conclusions

1. Classical & quantum dissipation rates computed in 2D billiards
 - first study of frequency-dependence in billiards
 - semiclassical correspondence found
 - applications: driven quantum dots, nuclei...
2. Class of ‘special’ deformations
 - friction coeff μ vanishes at dc
 - predicts new power laws $\mu(\omega) \sim \omega^\gamma$
 - dilation (*new*) \rightarrow eigenstates quasi-orthogonal on boundary
(*semiclassical* reason for Vergini-Saraceno method success)
3. Systematic subtraction of ‘special’ components of general $D(\mathbf{s})$
 - improved upon 25-year-old ‘wall formula’