

V63.0140-3: Linear Algebra. QUIZ 4

Tues 11/25/03. Please answer on this sheet, and write your name at the top. Check your working, and give explanations wherever you can.

1. Given the matrix $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$,

(a) Find the eigenvalues:

(b) Find the eigenvectors:

$$\mathbf{v} = \begin{bmatrix} \\ \end{bmatrix}, \begin{bmatrix} \\ \end{bmatrix}$$

(c) Write A in the form PDP^{-1} , where D is diagonal:

$$A = \begin{bmatrix} \\ \end{bmatrix} \begin{bmatrix} \\ \end{bmatrix} \begin{bmatrix} \\ \end{bmatrix}$$

2. (a) If a matrix has the characteristic equation $(\lambda-1)(\lambda+2)(\lambda+2) = 0$, can it be diagonalizable? Why?
- (b) Find the unit vector (*i.e.* length 1) in the direction of $\begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$.
- (c) True/false? If B has all eigenvalues distinct then B must *always* be diagonalizable.
- (d) The matrix $C = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$ has an eigenvalue $\lambda = 2 + 3i$. Find the eigenvector associated with this eigenvalue:

$$\mathbf{v} = \begin{bmatrix} \\ \end{bmatrix}$$

- (e) For the dynamical system $\mathbf{x}_{k+1} = A\mathbf{x}_k$ using A from Question 1, is the origin an attractor, repeller, or saddle-point?
3. Given the vectors in \mathbb{R}^2 , $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, and $\mathbf{y} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$,
- (a) Are \mathbf{u} and \mathbf{y} orthogonal? Why?
- (b) What is the orthogonal projection of \mathbf{y} onto \mathbf{u} ? [Hint: your answer should be a vector!]