

## V63.0140-3: Linear Algebra. QUIZ 3

Tues 11/4/03. Please answer on this sheet, and write your name at the top. Check your working, and give explanations wherever you can.

1. The matrix  $A$  has been converted to reduced echelon form as follows

$$A = \begin{bmatrix} -2 & -4 & 1 & 0 & 4 \\ 0 & 0 & 0 & 3 & 3 \\ 1 & 2 & 2 & 1 & 5 \\ 3 & 6 & 0 & -2 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Write down a basis for the column space of  $A$ :

Write down a basis for the null space of  $A$ :

What is  $\dim \text{Nul } A$ ?

What is  $\text{rank } A$ ?

What is the dimension of the subspace formed by the span of the rows of  $A$ ?

2. (a) True/false?  $\mathbb{R}^2$  is a subspace of  $\mathbb{R}^3$ ? Why?
- (b) True/false? The null space of  $A$  from question 1 is a subspace of  $\mathbb{R}^5$ ?
- (c) True/false? The column space of  $A$  from question 1 is isomorphic to  $\mathbb{R}^4$ ?
- (d) Does the set of functions  $\{2 + 4t^2, t + 3t^2, t + 2t^2\}$  form a basis for the polynomials  $\mathbb{P}_2$ ? If so, explain the criteria which you tested. If not, what dimension subspace do they span?

- (e) Does the set  $H = \left\{ \begin{bmatrix} 3a \\ 2a - 1 \end{bmatrix}, a \text{ is real} \right\}$  form a subspace of  $\mathbb{R}^2$ ? If so, what dimension is it? If not, name a test which failed.

3. Say  $H$  is some subspace of  $\mathbb{R}^3$ , for which a basis has been found to be  $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ . Find  $[\mathbf{x}]_{\mathcal{B}}$ , the coordinates of the point  $\mathbf{x} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}$  in the  $\mathcal{B}$  basis. (PS you should find that  $\mathbf{x}$  is in  $H$ ).

$$[\mathbf{x}]_{\mathcal{B}} =$$