V63.0140-3: Linear Algebra. QUIZ 2

Tues 9/30/03. Please answer on this sheet, and write your name at the top. Check your working, and give explanations wherever you can.

1. For A $= \begin{bmatrix} 5 & 10 \\ 2 & 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 15 \\ 6 \end{bmatrix}$, write the solution set of $A\mathbf{x} = \mathbf{b}$ in *parametric form*:

2. Are the three vectors $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 7 \\ 0 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ -1 \\ 1 \\ 1 \end{bmatrix}$ linearly independent?

   If not, write a linear dependence relation.

3. (a) True/false? If matrices A and B 'commute' this implies that $AB - BA = 0$.

   (b) True/false? If the range of a transformation T does not fill the codomain then T must not be one-to-one.

   (c) If a linear transformation takes $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} -5 \\ -1 \end{bmatrix}$, to what vector does it take $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$? Explain why using any relevant definition or theorem.

   (d) Is the matrix $\begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$ invertible?

4. A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ rotates points by angle $-\pi/2$ (*i.e.* , 90° *clockwise*). Find the *standard matrix* for T:

   Is T 'onto' $\mathbb{R}^2$?