V63.0140-3: Linear Algebra. Homework 10

due Thurs Nov 20 at start of lecture

5.4:

1. 3. 6. 14. 23.

5.5:

1.

A1) Show using complex arithmetic that for the eigenvalue $\lambda = 2 + i$ found in the previous question, the complex vectors $A\mathbf{v}_1$ and $\lambda_1\mathbf{v}_1$ are the same. ($\mathbf{v}_1$ is the corresponding eigenvector you found).

6. [Check your signs]

A2) In lecture we found the $\pi/2$-rotation matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ had $\lambda_1 = i$ with $\mathbf{v}_1 = (1, -i)$, and $\lambda_2 = -i$ with $\mathbf{v}_2 = (1, i)$.

a Show how the real vector $\mathbf{x}_0 = (1, 0)$ can be written as a lin. comb. of $\mathbf{v}_1$ and $\mathbf{v}_2$.

b Multiply this lin. comb. from the left by A, use the eigenvalues to show how the changing weights gives the real vector $A\mathbf{x}_0$.

c [Bonus] Why does $A^k = I$ imply that all eigenvalues obey $\lambda^k = 1$? (Above we had $k = 4$).

5.6:

1. (write part b in the form $\mathbf{x}_k$ = lin. comb. of $\mathbf{v}_1$ and $\mathbf{v}_2$ with weights being functions of k).

7.

12.

6.1:

2.

8.

12.

14.

16.

19.

24. (Prove it using algebra, definition of $||\mathbf{x}||$, etc).