V63.0140-3: Linear Algebra. REVIEW QUESTIONS

Some from book, some others to think about.

Studying: go over Quiz questions (all 4 quizzes, since you need to be secure on basics of quiz 1,2). Make sure you can redo quiz questions. Then go over HW, redoing ones you got wrong (excluding the very hardest ones, unless you have time). Then do some practice problems from below (answers are in text form on website). Make sure you spread your effort across most major topics.

Things I *didn't* focus on in Review session (due to time constraint) that are important to review: diagonalization (big one), Markov chains, least squares, orthogonal basis sets, quadratic forms.

Practise Questions from book

"PP" means Practise Problem, which have worked solutions after the Exercises.
4.1: PP1, 9, 14, 17.
4.2: 15, 23.
4.3: 11, 13.
4.4: 1, 5, 21.
4.5: 11, 13.
4.6: PP1-4, 1 (good summary).
4.7: 5, 9.
4.9: 11 (b is asking for long-time limit)
Ch. 4 Supplementary Exercises: 1 (good but some are hard), 7 (use Rank Theorem).
5.1: 1, 9, 17, 23, 25.
5.2: 3, 11, 20.
5.3: 3, 5, 17, 25.
5.4: 7, 9, 11, 13.
5.5: 5, 21.
5.6: 2, 8.
Ch. 5 Supplementary Exercises: 1, 2.

6.1: 20, 25.
6.2: 11, 13, 17, 27.
6.3: 7, 11, 17.
6.4: 3, 9.
6.5: 1, 13, 17.
Ch. 6 Supplementary Exercises: 1, 5.
Repeated from what I gave in class:
7.1: 7, 9, 13, 19, 25a-c.
7.2: 1, 3, 7, 9, 11.

Other questions

Note these are biased towards early material. Don't forget to practise some later stuff too!

1. How does the solution set to $A\mathbf{x} = \mathbf{b}$ relate to the solution set of $A\mathbf{x} = \mathbf{0}$? (Pre-midterm question, but an important reminder).

2. In review we showed $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : \begin{array}{rcl} x+y-z & = & 0 \\ 2y+z & = & 0 \\ -x-3y & = & 0 \end{array} \right\}$ was a subspace because it could be written as Nul A for some A. Find a basis for W. What is dim W ?

3. Is $B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$ a basis for $W = \text{Span} \left\{ \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} \right\}$?

4. Is $B = \left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \end{bmatrix} \right\}$ a basis for $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x + 2y = 0 \right\}$?

5. Is $B = \{1+t, t+t^2, 1+t^2\}$ a basis for $\mathbb{P}_2$, the space of all 2nd-order polynomials?

6. If A is a 6-by-8 matrix, and dim Nul $A = 3$, what is rank A ?

7. Can a system of 3 linear equations in 5 unknowns be unique? If not, is the solution set always a subspace?

8. V is a finite dimensional vector space. Let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ be vectors in V. Define linear independence of this setof vectors. (pre-midterm review).

9. Define isomorphism.

10. Let $\mathbf{b} = (1, 5, 7, 0)$, with the solution to $A\mathbf{x} = \mathbf{b}$ being unique. You have not been given the size of A. If dim Row $A = 3$, what condition on the number of columns of A holds? Do all $\mathbf{b}$ lead to a consistent set of equations?

11. For an $n \times n$ matrix, why does the property that all n eigenvalues are distinct result in the matrix being diagonalizable?

12. What condition on k holds if the matrix $A = \begin{bmatrix} 3 & 0 \\ k & 3 \end{bmatrix}$ is diagonalizable?

13. For any real vector $\mathbf{u}$, why is $\mathbf{u} \cdot \mathbf{u}$ never negative?

14. Define the orthogonal complement of a subspace W.

15. In which direction does $\mathbf{x}_k$ point in the long-time limit $k \to \infty$ for the discrete dynamical system $\mathbf{x}_{k+1} = A\mathbf{x}_k$? Why?

16. What condition on A holds if the least-squares solution to $A\mathbf{x} = \mathbf{b}$ is unique?