V63.0140-3: Linear Algebra. Solution to WORKSHEET on chemical balancing.

from Lecture 4, date 9/11/03, part B of worksheet

We want to find *nontrivial* (x_1, x_2, x_3, x_4) that make

$$x_1\, C_2H_6O + x_2\, O_2 \longrightarrow x_3\, H_2O + x_4\, CO_2$$

a balanced chemical equation.

C_2H_6O has two C, six H, one O, so gives the vector $(2, 6, 1)$. Similar for other molecules.

Vector equation

$$x_1 \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

Rearrange to standard form

$$x_1 \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

It's homogeneous. We can write the augmented matrix (whose last column is therefore zero),

$$A = \begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 6 & 0 & -2 & 0 & 0 \\ 1 & 2 & -1 & -2 & 0 \end{bmatrix}$$

and reduce it to Reduced Echelon Form, seeing x_4 is the only free variable,

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \end{bmatrix}$$

General solution has the free parameter x_4. Make sure you understand how to read off the vector form of this solution from the above R.E.F. matrix, like we did today in class:

$$\mathbf{x} \equiv \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x_4 \\ \frac{3}{2}x_4 \\ \frac{3}{2}x_4 \\ x_4 \end{bmatrix} = x_4 \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \\ \frac{3}{2} \\ 1 \end{bmatrix}.$$

We can write the solution set as $\mathbf{x} = t\mathbf{v}$, where $\mathbf{v} = (1, 3, 3, 2)$ and t is just some parameter. (Note we multiplied the entries in the vector to make them all integers).

Therefore the balanced chemical equation is

$$1\ C_2H_6O + 3\ O_2 \longrightarrow 3\ H_2O + 2\ CO_2$$

You can check the total number of each C, H, O work out. (For such simple cases you could have done it 'by hand' faster...).