

V63.0140-3: Linear Algebra. Solution to WORKSHEET on chemical balancing.

from Lecture 4, date 9/11/03, part B of worksheet

We want to find *nontrivial* (x_1, x_2, x_3, x_4) that make



a balanced chemical equation.

C_2H_6O has two C, six H, one O, so gives the vector $(2, 6, 1)$. Similar for other molecules.

Vector equation

$$x_1 \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

Rearrange to standard form

$$x_1 \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

It's homogeneous. We can write the augmented matrix (whose last column is therefore zero),

$$A = \begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 6 & 0 & -2 & 0 & 0 \\ 1 & 2 & -1 & -2 & 0 \end{bmatrix}$$

and reduce it to Reduced Echelon Form, seeing x_4 is the only free variable,

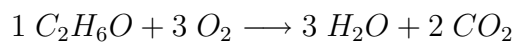
$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \end{bmatrix}$$

General solution has the free parameter x_4 . Make sure you understand how to read off the vector form of this solution from the above R.E.F. matrix, like we did today in class:

$$\mathbf{x} \equiv \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x_4 \\ \frac{3}{2}x_4 \\ \frac{3}{2}x_4 \\ x_4 \end{bmatrix} = x_4 \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \\ \frac{3}{2} \\ 1 \end{bmatrix}.$$

We can write the solution set as $\mathbf{x} = t\mathbf{v}$, where $\mathbf{v} = (1, 3, 3, 2)$ and t is just some parameter. (Note we multiplied the entries in the vector to make them all integers).

Therefore the balanced chemical equation is



You can check the total number of each C, H, O work out. (For such simple cases you could have done it 'by hand' faster...).