V63.0123-1 : Calculus III

Instructor: Alex Barnett

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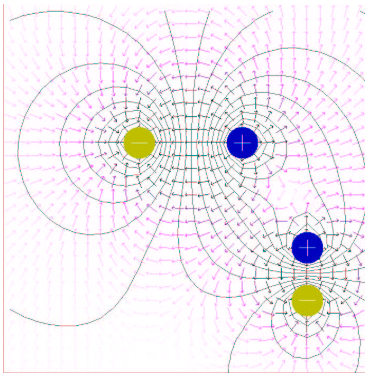
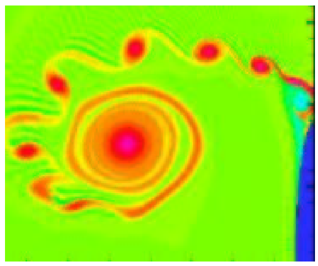
The intersection of the cone with the sphere





In this course we generalize ideas of differential and integral calculus to two and three dimensions. As well as giving us tools to model the real, three-dimensional world through physics, chemistry, engineering, etc, this is also essential to understanding functions of many variables in more abstract situations, such as arise in computer science, economics, and statistics and probability. The emphasis will be on intuition, visualization, and problem-solving, rather than theorem-lemma-proof. I am a physicist by training and I am sure this will show.

Lectures: Waverly 435, Mondays and Wednesdays, 8:55am – 10:45am. I plan on 2×50 mins, with 10 min break. There may be small-group activities.

Required book: *Multivariable Calculus* by James Stewart, 4th Ed. (Brooks/Cole 1999). About \$95 new, \$50 second-hand (*e.g.* Amazon.com).

If you benefit from seeing lots of the solutions worked through, consider: *Student Solutions Manual for Multivariable Calculus* by Dan Clegg and Barbara

Frank (Brooks/Cole 1999). About $35.

Web-site: I will post homeworks, useful material, applets that I find, etc, at `http://www.cims.nyu.edu/~barnett/calc3/`

Grades: There will be 2 midterms and 1 final. Your grade will be determined from whichever of these two weighting schemes is better for you:

- homework 25%, midterms 35%, final 40% (for exam-lovers)
- homework 35%, midterms 30%, final 35% (for homework-lovers)

We will discuss options for take-home vs in-class midterms.

Homework: 12 problem sets, mostly from the book, due Wednesday lectures. Permission to hand in late must be obtained the previous week, and otherwise, unless there are exceptional circumstances, the homework's grade will be multiplied by 0.5 for each day it is late. Collaboration is encouraged, but writeups must be done alone. Only the best 10 homeworks grades will be used (*i.e.* the lowest 2 dropped).

Office Hours: Tuesday 10am–noon (rm 1122 WWH).

Tutoring: Available 4 days a week on a drop-in basis at rm 704, WWH. Hours seem to be 1pm–8pm Mon+Wed, 11am–8pm Tues+Thurs. Tutors may vary in their knowledge of multivariable calculus. If you find a particularly good tutor for Calc III material, let me know and I will share with the class.

Timeline: (approximate)

wk		date	due	book	content
1	W	Jan 22		11.1-11.3	parametric curves in 2d.
	M	27			arc length, polar coords
2	W	29	HW1	13.4-13.6	3d vectors, cross product
	M	Feb 3			planes and cylinders in 3d
3	W	5	HW2	14.1-14.4	intro to vector calculus
	M	10			normals, curvature, kinematics
4	W	12	HW3	15.1-15.3	functions of several variables
		Feb 17 : Presidents' Day Holiday			
5	W	19	Midterm1		partial derivative
	M	24		15.4-15.5	linear approximation, chain rule
6	W	26	HW4	15.6	grad
	M	Mar 3		15.7	minima and maxima, higher partials
7	W	5	HW5	15.8	Lagrange multipliers
	M	10		16.1-16.2	double integrals
8	W	12	HW6	16.3-16.5	more double integrals
		Mar 17–22 : Spring recess			
	M	24			(in polar coords, applications)
9	W	26	HW7	16.6-16.7	surface area, triple integrals
	M	31		16.8	(in cylindrical, spherical coords)
10	W	Apr 2	HW8	16.9, 17.1	change of variables, vector fields
	M	7	Midterm2	17.2	intro to line integrals
11	W	9	HW9	17.3	line integrals of vector fields
	M	14		17.4	Green's theorem in 2d
12	W	16	HW10	17.5-17.7	curl and div, parametric surfaces
	M	21			surface integrals of vector fields
13	W	23	HW11	17.8-17.9	Stokes' theorem
	M	28			divergence theorem
14	W	30	HW12		review
	M	May 5			review
		May 7–14 : Finals			

Conceptual Outline: *(with a few example applications)*

- Week 1 : reminder of thinking in 2d. Parametric curves are just 2d vector-valued functions of one variable, $\mathbf{r}(t)$ where $\mathbf{r} \equiv (x, y)$.
- Week 2 : 3d vectors $\mathbf{r}$. *(force vectors, torque, magnetic force).*
- Week 3 : 3d vector-valued functions of 1 variable, $\mathbf{r}(t)$, where $\mathbf{r} \equiv (x, y, z)$. *(motion of a particle, velocity, acceleration).*
- Weeks 4–9: scalar functions of many variables, $a(\mathbf{r})$, also called a 'scalar field'.
  - (Weeks 4–6) How to take derivatives in various directions. Combining them into a vector function called 'grad a'. *(heat conduction).*
  - (Weeks 7–9) How to integrate such a function over various regions of space. *(engineering: calculation of volumes. chemistry & materials science: calculate overlaps of electron quantum wavefunctions).*
- Weeks 10–12: vector-valued functions of many variables, $\mathbf{u}(\mathbf{r})$, also called a 'vector field'. Integrating $\mathbf{u}$ along a line in 3d *(computing a voltage difference).* Derivatives 'div $\mathbf{u}$' and 'curl $\mathbf{u}$'. *(fluid motion, vortices, sources and sinks).*
- Week 13: Climax: two amazing & useful geometric theorems.
  - Divergence theorem: the integral of div $\mathbf{u}$ in a volume is equal to a 'surface integral' of $\mathbf{u}$. *(foundation of electrostatics, conservation of mass in fluid flow),*
  - Stokes theorem: the surface integral of curl $\mathbf{u}$ is equal to the 'line integral' of $\mathbf{u}$ around the edge of the surface. *(electromagnetic induction).*

Coordinate systems:

- Generally we start in the Cartesian (x, y, z) coordinate system.
- Multivariable integration will also be handled in other systems: polar (r, θ), cylindrical (r, θ, z), spherical (r, θ, ϕ).
- Vector products (dot, cross), and vector derivatives (grad, div, curl) will be handled only in Cartesian system.