V63.0123-1 : Calculus III. Mini-Quiz Solutions

Wed 22 Jan

a) evaluate $\frac{d}{dx}\frac{1}{\tan x}$:
You don't need to do substitution here (would only for integrals):

$$\frac{d}{dx}\left(\frac{1}{\sin x}\cos x\right) \overset{\text{product rule}}{=} \frac{1}{\sin x}\frac{d}{dx}\cos x + (\cos x)\frac{d}{dx}\frac{1}{\sin x} = \frac{-\sin x}{\sin x} - \frac{\cos^2 x}{\sin^2 x}$$
$$\overset{\text{trig ident}}{=} -(1+\cot^2 x) = -\text{cosec}^2 x \quad \text{or} \quad -\frac{1}{\sin^2 x}.$$

b) evaluate $\frac{d}{dx}\sin(\sqrt{x}+1)$:

$$\overset{\text{chain rule}}{=} \left(\frac{d}{dx}(\sqrt{x}+1)\right)\cos(\sqrt{x}+1) = \frac{1}{2\sqrt{x}}\cos(\sqrt{x}+1)$$

c) evaluate $\int_0^{\pi/2} d\theta\,(\theta+\sin 2\theta)$:
Sum of 2 separate integrals, remember integral of $\sin(\alpha\theta)$ is $-\frac{1}{\alpha}\cos(\alpha\theta)$:

$$\begin{aligned}\text{answer} &= \left[\tfrac{1}{2}\theta^2\right]_0^{\pi/2} - \left[\tfrac{1}{2}\cos 2\theta\right]_0^{\pi/2} = \tfrac{1}{2}\cdot(\pi/2)^2 - \tfrac{1}{2}(\cos\pi - \cos 0)\\ &= \frac{\pi^2}{8} + 1.\end{aligned}$$

You need to become very familiar with values of sin, cos and tan.

d) will be done in lecture.