V63.0123-1 : Calculus III. Homework 6

due Wed Mar 12 at lecture. Continued on back side!

(Review finding extrema with two variables)

A1. A triangle is formed by the points $(a, 0)$, (b, c), and the origin. Holding the triangle's area constant at $\sqrt{3}$, find values of a, b and c that minimize L, the sum of the squares of the lengths of the three sides. What is special about the triangle(s) that you have found?

Hints:

Draw a diagram showing a, b, c and the triangle.

Write a formula for the area, and a formula for L (check it).

Use the fact that the area is constant to eliminate one variable.

Write an expression for L in terms of the other two variables, simplify it, then minimize it.

(Review gradient vector and regular single-variable chain rule)

A2. Imagine you have a general function $f(x, y, z)$. You now make a general function of this, namely $h(f)$. We want to consider h as a function of (x, y, z).

a) Draw the 'tree of dependence' for this situation (it will look simpler than those in section 15.5)

b) Calculate ∇h in terms of ∇f. [Hint: components].

c) What can we say about the direction of ∇h compared to that of ∇f ?

d) Use this to explain why if you want to find a point where some distance is minimized, then minimizing the *squared* distance (which is an easier formula) gives the same answer.

15.8: (Lagrange multipliers)

1. The question asks for absolute minimum and maximum. Please also state how many *local* minima and maxima there are on the curve.

3.

4.

18. Remember to find both absolute minimum and maximum, and to look in both interior and on the boundary curve. Sketch the region and the locations of the extrema.

A3. Repeat problem A1 but using Lagrange multipliers, treating the area and L as functions of the 3 variables (a, b, c). Is it easier?

16.1: (Double integrals over rectangles)

4. First draw a diagram of your rectangle and subrectangles.

12.

18.

16.2: (Iterated integrals)

2.

4.

8.