

Mathematics at Dartmouth



August 2007

Cover photo: The first floor of Kemeny Hall; former math building Bradley Hall (now demolished) can be seen through the window.

Photographs by Joseph Mehling, Dartmouth College Photographer

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Introduction

Dartmouth College has an outstanding mathematics department devoted to excellence of instruction and scholarship. Faculty members have research and teaching interests not only in the major areas of pure mathematics, but also in applications of mathematics to the physical, biological, and social sciences. The undergraduate program in mathematics is among the finest in the country. It provides a level of individual attention one would expect only at a four-year college along with the kind of research activity found almost exclusively at major universities. The department has long had a reputation for both high quality teaching and successful innovation in education.

The department sponsors a Ph.D. program designed to develop strong research mathematicians who are excellent teachers. The demand for graduates of this program by institutions that prize good teaching is strong evidence of its success. Graduate students enjoy close personal contact with faculty, the added dimension of interaction with other students, and the mutually satisfying experience of working with Dartmouth undergraduates.

A Faculty Strong in Teaching and Research

The Mathematics Department prides itself on a faculty as strong in teaching as in research. A faculty engaged in both undergraduate and graduate teaching is necessarily more alert and responsive to change in both areas. Educational reforms pioneered by the department have been widely copied throughout the nation, and new ideas and programs are constantly under consideration. Following is a list of permanent members of the faculty with Ph.D.-granting institution and major research interest.

Department Faculty

Martin Arkowitz Cornell University; Algebraic topology

Alex Barnett Harvard University; PDE, Numerical analysis, Applied math

Vladimir Chernov Uppsala; Manifolds and cell complexes, Global analysis, analysis on manifolds, Differential geometry

Peter Doyle Dartmouth College; Geometry, Probability



Carson Commons, Kemeny Hall's faculty lounge (L-R: Jody Trout, Scott Pauls, Stephanie Treneer, Craig Sutton).

Sergi Elizalde MIT; Combinatorics

Carolyn Gordon Washington University; Differential geometry, Lie groups

Marcia J. Groszek Harvard University; Mathematical logic, Set theory

C. Dwight Lahr Syracuse University; Analysis

Rosa Orellana University of California–San Diego; Combinatorics

Scott Pauls University of Pennsylvania; Differential geometry, Calculus of variations and optimal control, Optimization

Carl Pomerance Harvard University; Number theory, Cryptology

Daniel Rockmore Harvard University; Analysis, Applied math, Signal/image processing, Mathematical finance

Thomas R. Shemanske University of Rochester; Number theory

Craig J. Sutton University of Michigan; Riemannian geometry, Lie groups

John Trout Penn State; Functional analysis, K-theory, Operator theory

Dorothy I. Wallace University of California–San Diego; Number theory

David Webb Cornell University; Differential geometry; K-theory

Rebecca Weber University of Notre Dame; Mathematical logic, Computability theory

Dana P. Williams University of California–Berkeley; Functional analysis, Abstract harmonic analysis, Topological groups, Lie groups

Peter Winkler Yale University; Combinatorics; Probability theory and stochastic processes; Computer science

Each year the permanent faculty is supplemented by several John Wesley Young Postdoctoral Fellows, who are also active in both teaching and research. Frequently one or two outstanding visiting professors spend all or part of the year as members of the mathematics faculty.

The size of Dartmouth College, with approximately 4100 undergraduates and 600 graduate students in the Arts and Sciences, is large enough to allow a faculty active in a wide variety of research work, but small enough to assure close contact between students and faculty.

The Mathematics Department has received numerous grants for research, education, and special projects from government and private foundations. These support the research of individual faculty members, undergraduate research activities, and development of innovative educational programs and other departmental endeavors.

Mathematical Community at Dartmouth

The Mathematics Department is housed in the beautiful new John Kemeny Building (completed 2006), which holds classrooms, seminar rooms, lounges, laboratories, and offices for faculty and graduate students. Undergraduates enjoy studying and networking in the attractive Undergraduate Commons on the first floor. On the second floor, one invariably sees groups of graduate students, frequently mixed with advanced undergraduates, enjoying a discussion of an intriguing mathematical problem. Located next door, the Wallace Cook Memorial Mathematics Collection in the Baker-Berry Library contains over 25,000 volumes. Students have convenient access to basic reference materials as well as to over 300 current periodicals in the mathematical sciences. The Dartmouth digital library includes over 425 online mathematics journals as well as numerous other resources.

The very active Dartmouth Mathematics Society and Dartmouth Student Chapter of the Association for Women in Mathematics welcome all students regardless of their level or major. The activities of the department facilitate informal contact among students, faculty, and the many visitors to the Department. Such visitors may report on recent research or give extracurricular talks to students.

The Undergraduate Program

To Stimulate and Challenge

In keeping with Dartmouth College's well-established commitment to liberal arts education, the Mathematics Department gives special attention not only to the future mathematician, scientist, and engineer, but also to students whose primary interests are in the humanities or social sciences. Dartmouth faculty recognize that knowledge of the mathematical sciences is important not only for understanding science, but also for a better grasp of many other disciplines.

A large majority of Dartmouth undergraduates take at least one course from the Mathematics Department during their college careers. These students naturally represent a wide variety of interests and degrees of preparation. The department takes great care to provide a corresponding variety in its course offerings. These offerings include honors courses for students strongly motivated in mathematics. Every student at Dartmouth has a computer and the department often integrates the use of the computer in its courses. There



Kemeny Hall's Undergraduate Commons
(L-R: Patrick Karas '08, Jessica Wheeler '10, Melinda Wilson '09,
Kiran Parkhe '07, Caroline Livett '07, Varun Jain '07).

are laboratories available to undergraduates with networks of computers running Macintosh, Windows and various UNIX/Linux operating systems, and the campus is blanketed by wireless internet access points.

Introductory Courses

Students interested in mathematics, engineering, or the sciences typically begin their studies with calculus. Students whose primary interests lie outside those areas have a variety of courses without prerequisites available.

Calculus

Dartmouth offers a three-term calculus sequence along with several alternate versions. The course "Applications of Calculus to Medicine and Biology" is especially well-suited for pre-med and biology students and may be taken after the first term of the calculus sequence.

For those students whose secondary school mathematics has not prepared them for the usual first-term calculus course, the Department offers a special

two-term sequence of courses which includes background material in algebra as well as introductory calculus. Enrollment in these courses requires departmental permission. Following completion of these two courses, students may enroll in the second course of the standard calculus sequence.

First year honors program

Students with exceptional interest and aptitude in mathematics are invited to enroll in honors sections of the introductory courses. In such courses, not only the mathematical tools are studied, but also the deeper structure underlying those tools. They provide a smaller class size than the regular courses and a sophisticated approach to the material. The honors sections attract students with wide ranging interests and anticipated majors.

Students who would like to sample abstract mathematics may enroll in “An Introduction to Mathematics Beyond Calculus” upon completion of (or with advanced placement credit for) the first two terms of the calculus sequence. This recently established special topics course is ideal for students contemplating a major in mathematics but unsure what the subject entails.

Innovative courses with no prerequisites

“Introduction to Finite Mathematics” and “Exploring Mathematics” both offer an introduction to mathematical topics outside calculus. Finite mathematics consists of sets and logic, probability, counting techniques, and topics in matrix algebra, and will be of particular interest to students in the social and biological sciences.

“Exploring Mathematics” is a special topics course; recent offerings include “Chance”, “Pattern”, “A Matter of Time”, and “The Mathematics of Music and Sound”. This course is aimed at beginning students who are not necessarily interested in pursuing further mathematics, and the specific topic is determined by the faculty member teaching the course each term it is offered. “A Matter of Time” explores historical approaches to measuring time, time in literature and film, and Einstein’s Theory of Relativity. “The Mathematics of Music and Sound” explores acoustics, waves, resonance, hearing, the workings of musical instruments, tuning and composition, and more, from a mathematical perspective.

Students interested in a career in business, but not contemplating a mathematics major, will find that the courses in finite mathematics, introductory calculus, and statistics provide a strong mathematical foundation.

Advanced placement

Many students receive one or more terms of advanced placement credit for calculus courses successfully completed prior to enrollment at Dartmouth. An undergraduate entering with such training may begin study at whatever level is deemed appropriate by the mathematics department. It is also possible to obtain advanced placement credit for ordinary differential equations, linear algebra, or other more advanced mathematics. Placement and credit is based on a combination of College Board achievement and SAT scores, College Board advanced placement test scores, and the result of a placement test administered on entering Dartmouth. Students who believe they are suited for honors courses or advanced placement are encouraged to discuss the matter with the Department's Advisor to First-Year Students or Advisor to Majors.

Mathematics Major

The major in mathematics is intended both for students who plan careers in mathematics and related fields, and for those who simply find mathematics interesting and wish to continue its study. Each year Dartmouth mathematics majors enter professional schools in business, medicine or law, as well as Ph.D. programs in pure and applied mathematics. Many enter the world of finance. The content of the major is quite flexible, and courses may be selected largely to reflect student interests. In addition to regular course offerings, a student with specialized interests not reflected in our current course offerings may arrange for an independent reading course. Proposals for independent activities should be directed to the Departmental Advisor to Mathematics Majors.

A detailed discussion of the major may be found in the Dartmouth College Bulletin of Organization, Regulations, and Courses (ORC) and at <http://math.dartmouth.edu/> online. Any student who considers majoring in mathematics should read the relevant material in this reference carefully. In brief, the major is organized as follows. After taking the major's prerequisites, a student must take eight upper level courses, including an abstract algebra course and an analysis course, and complete a culminating experience. Many of the advanced undergraduate courses fulfill the culminating experience require-



One-on-one interaction with faculty is common at Dartmouth
(L-R: Rebecca Weber, Jonathan Gurwitz).

ment; other options for the culminating experience include an honors thesis, a graduate level mathematics course, or a one-term research project (subject to departmental approval). Although a student can craft a major following only the basic requirements, the department provides three collections of recommendations: one emphasizing pure mathematics, one emphasizing applied mathematics, and one in mathematics education. The specific course suggestions may be found in the ORC.

The Mathematics Department offers an unusually large array of courses to prepare students for graduate study in pure or applied mathematics. Advanced undergraduate students who have taken the prerequisites courses may also enroll in graduate classes. The study of pure mathematics is centered on algebra, analysis, and topology, with opportunities to pursue many additional subjects such as combinatorics, logic, geometry, and number theory. The department has recently expanded the variety of applied mathematics courses available, including introductory and topics courses in general applied math-

ematics. A pair of financial mathematics courses will be offered beginning in the academic year 2007–08.

All mathematics majors, whatever their interests, are encouraged to take linear algebra, Mathematics 22/24, as soon as is feasible. Not only is this course an explicit prerequisite to many upper-division courses, but also the level of mathematical sophistication developed in Mathematics 22/24 will be presumed in many upper-division courses for which linear algebra is not an explicit prerequisite.

Mathematics Minor

In addition to a major and modified major, the Department also offers several minors in mathematics. At present, the Department allows a student to obtain a minor in mathematics, applied mathematics for physical and engineering sciences, and in applied mathematics for biological and social sciences. Details are given in the Dartmouth College Bulletin of Organization, Regulations, and Courses and at <http://math.dartmouth.edu/> online.

Advanced Undergraduate Courses

The courses listed below are (with occasional exceptions) offered every year or every other year.

17. An Introduction to Mathematics Beyond Calculus
19. Discrete Mathematics in Computer Science
20. Discrete Probability
22. Linear Algebra with Applications
23. Differential Equations
24. Linear Algebra (Honors Section of Mathematics 22)
25. Number Theory
26. Numerical Methods in Computation
27. Advanced Calculus and Dynamics in Biology and Medicine

- 28. Introduction to Combinatorics
- 29. Introduction to Computability
- 31. Topics in Algebra
- 32. The Shape of Space
- 33. Mathematics in the Sciences and Engineering
- 35. Real Analysis
- 36. Mathematical Models in the Social Sciences
- 38. Graph Theory
- 39. Logic
- 40. Topics in Applied Probability
- 42. Differential Geometry I
- 43. Functions of a Complex Variable
- 46. Introduction to Applied Mathematics
- 50. Probability and Statistical Inference
- 53. Chaos!
- 54. Topology I
- 56. Numerical Analysis
- 60. Probability (Honors Section of Mathematics 20)
- 63. Real Analysis (Honors Section of Mathematics 35)
- 66. Mathematical Topics in Modern Physics
- 68. Algebraic Combinatorics
- 69. Logic (Honors Section of Mathematics 39)
- 71. Algebra (Honors Section of Mathematics 31)

- 72. Topics in Geometry (Honors Section of Mathematics 42)
- 73. Multivariable Analysis
- 74. Topics in Topology
- 75. Applied Topics in Number Theory and Algebra
- 76. Topics in Applied Mathematics
- 81. Abstract Algebra
- 87. Reading Course
- 88. Special Topics
- 89. Seminar in Logic
- 97. Undergraduate Research
- 98. Senior Seminar

The Graduate Program

Dartmouth College offers a program of graduate study leading to the Ph.D. degree in mathematics. This program is designed to produce mathematicians who are highly qualified in both teaching and research. The College provides an environment in which doctoral candidates with superior abilities can pursue their professional study in mathematics and broaden their interests in mathematical teaching and learning. There are typically twenty to thirty graduate students in mathematics, so classes tend to be small and informal. In addition, since faculty members are quite accessible, there is a great deal of out-of-class interaction between faculty and graduate students. In order to achieve both breadth and depth in mathematics, students are expected to be engaged in full-time study including course work, seminars, research, and teaching.

Students who satisfactorily complete certification requirements in four areas of mathematics are normally awarded the Master of Arts (A.M.) degree. Those who are then admitted to Ph.D. candidacy and fulfill the remaining requirements, including the preparation and defense of a doctoral dissertation,

are awarded the Ph.D. degree. The dissertation consists of original results in the theory or application of some branch of mathematics. Areas in which students have recently chosen to specialize for their dissertation research include algebra, analysis, topology, applied mathematics (e.g., cryptology, numerical analysis, signal processing), mathematical biology, combinatorics, geometry, logic, number theory, and set theory.

The Ph.D. program in mathematics at Dartmouth has a strong reputation for preparing its students to be effective teachers as well as researchers. The Department is committed to helping its graduate students develop as teachers by example, by instruction, and by provision of an opportunity to gain realistic teaching experiences.

Recipients of the Ph.D. degree from Dartmouth have found employment at a broad cross section of academic institutions, including major universities and outstanding 4-year liberal arts colleges. Some are now working as research mathematicians in industry and government as well.



Hallway alcoves with blackboards encourage collaboration (L-R: Lizz Moseman, Mitsuo Kobayashi, Rachel Esselstein, Nick Scoville).

Requirements for the Ph.D. Degree

The requirements for the Ph.D. degree in mathematics are as follows:

1. Departmental certification in algebra, analysis, topology, and one other area of the student's choosing. Certifications must be completed within deadlines set by the department. The certification process is discussed below.
2. Admission to Ph.D. candidacy by the departmental Graduate Program Committee as a result of a review at the end of the spring term of the student's second year. This review will take into account all of the relevant information the Graduate Program Committee can gather, such as the student's performance during the certification process and an estimate of the student's ability to fulfill the remaining degree requirements and to write an acceptable thesis.
3. Demonstration of a reading knowledge of one foreign language, normally chosen from French, German, and Russian. This demonstration is arranged with a faculty member approved by the department and should take place by the end of the winter term of the student's fourth year.
4. Completion of a doctoral thesis of acceptable quality, and its defense in an oral presentation to the faculty.
5. Preparation for the teaching seminar through such activities as tutoring in the years before admission to candidacy, completion of the teaching seminar, and successful teaching of two courses or the equivalent.

Requirements for the Master's Degree

With rare exceptions, the A.M. in mathematics is offered only to those enrolled in the Ph.D. program. Normally the requirements for the A.M. must be fulfilled within two years of enrollment as a graduate student in the Dartmouth Mathematics Department (continuation in the program for a second year is contingent on a review of the student's work by the Graduate Program Committee, the review to take place early in the spring term of the first year). The departmental requirements for the A.M. are certification in algebra, analysis, topology, and one other area. This is in addition to the College requirements

for the Master's degree: three terms in residence at Dartmouth and credit in eight courses of graduate quality (these courses may sometimes, up to a limit of four, be replaced by approved research or special study).

The Certification Process

A student is considered certified in an area of mathematics when two faculty members with expertise in that area attest that the student has learned the material on the departmental syllabus for that area. The faculty members base their evaluation on course work, supervised independent study, informal discussions, and any other means that seem appropriate. Usually the certification process includes an informal oral examination. Printed departmental syllabi in algebra, analysis, and topology, as well as in frequently chosen fourth areas, are available on the Mathematics Department's web site. If the student desires certification in an area in which no departmental syllabus exists, the student must consult the Graduate Program Committee before making the choice; if the Committee concurs in the appropriateness of the choice, it will arrange for a syllabus to be prepared. Fourth areas of certification recently chosen by graduate students have included combinatorics, geometry, logic, probability, number theory, and applied mathematics.

Graduate Student Teaching

The Dartmouth Mathematics Graduate Program places a strong emphasis on preparing its students to be effective teachers. This preparation occurs in several stages. In the first two years in residence, students typically serve two terms per year as teaching assistants, conducting tutorials (or perhaps small discussion sessions) for elementary courses.

Following admission to candidacy at the end of their second year, students participate in an intensive ten-week summer course on the learning and teaching of mathematics. Subsequently, the student is typically given supervised responsibility for teaching one full ten-week course during each of the third and fourth years in residence. Graduate students have a faculty consultant available to give advice for each full course taught.

In consultation with the student and advisor, each fifth year graduate student is assigned one of a variety of responsibilities to further enhance his or her teaching preparation.



The graduate lounge provides a comfortable location for collaboration or a break from work (L-R: Naomi Davis, Lizz Moseman, Geoff Goehle).

Graduate Courses

Mathematics courses numbered 60 and above (listed earlier in this booklet) are undergraduate honors courses that may also be taken by graduate students for graduate credit, as may Mathematics 54. Courses numbered 100 and above are graduate courses, although advanced undergraduates may also enroll in those labeled 100–129, with the exception of 127.

- 100. Topics in Probability Theory
- 101. Topics in Algebra
- 102. Topics in Geometry
- 103. Topics in Analysis
- 104. Topics in Topology

- 105. Topics in Number Theory
- 106. Topics in Applied Mathematics
- 107. Supervised Tutoring
- 108. Topics in Combinatorics
- 109. Topics in Mathematical Logic
- 110. Probability Theory
- 111. Algebra
- 112. Geometry
- 113. Analysis
- 114. Algebraic Topology
- 115. Number Theory
- 116. Applied Mathematics
- 118. Combinatorics
- 119. Mathematical Logic
- 120. Current Problems in Probability Theory
- 121. Current Problems in Algebra
- 122. Differential Geometry
- 123. Current Problems in Analysis
- 124. Current Problems in Topology
- 125. Current Problems in Number Theory
- 126. Current Problems in Applied Mathematics
- 127. Supervised Reading Course
- 128. Current Problems in Combinatorics

- 129. Current Problems in Mathematical Logic
- 137. Independent Reading
- 147. Teaching Seminar
- 148. Independent Project
- 149. Supervised Teaching
- 156. Graduate Research
- 157. Thesis Research
- 158. Independent Research

First-year graduate students typically enroll in the undergraduate courses Math 73 (Multivariable Analysis) and Math 81 (Abstract Algebra) unless their undergraduate preparation included the equivalents of these courses. Approximately twelve of the courses numbered 100 and above (in addition to individual reading, research, tutoring and teaching) are offered each year. Six foundational courses in the areas of algebra, analysis, and topology are offered yearly and are directed towards the first- and second-year graduate students. The areas and topics of the remaining courses vary from year to year according to the interests of the graduate students and faculty members. Graduate students normally enroll in three courses during each term, so they find variety in both course content and sophistication as they choose their courses.

Advanced graduate students may be granted credit for independent reading in topics of special interest. In addition, there are series of department seminars in algebra, analysis, combinatorics, topology, applied mathematics, number theory, geometry, and logic, as well as frequent seminar series in other specialties or in interdisciplinary topics. These seminars form an important component of the education of graduate students.

Fellowships

Fellowships are usually awarded to all entering Dartmouth mathematics graduate students. As of Fall 2006, these awards are \$1800 per month, continuing year round from the time the student begins study until the time the student has graduated, up to four years, with a fifth-year extension upon approval by

the Graduate School. As a result of changes in the cost of living, in the past the stipend has been adjusted upwards annually; it is the intention of the College to continue to make adjustments as circumstances permit. All awards include a supplementary scholarship covering tuition costs. Since all applicants for admission are considered for fellowships, no special fellowship application is required.

Admission

Applicants should submit online a completed application form, undergraduate grade transcript, three letters of recommendation from persons qualified to evaluate their potential as scholar and teacher, and a list of textbooks used in upper level mathematics courses. No application fee is required. Information and the application form may be found at <http://math.dartmouth.edu/> and inquiries may also be made by telephone (603-646-3722) or by electronic mail (mathphd@dartmouth.edu).

Applicants must take both the General and the Mathematics subject test of the Graduate Record Examination and have the results sent to the department. Scores will reach the department in time to be useful if the tests are taken by December of the senior year. Students for whom English is a second language must take all parts of the TOEFL Examination and arrange for the scores to be sent to Dartmouth. Students must be able to use English as teaching fellows on arrival. **We do not waive these requirements.** Admission decisions can be made only on the basis of official score reports issued to the department and not on the basis of copies of the student's score reports.

Applications should be completed as soon as possible and, in any case, by February 15. Admissions and fellowship awards will be announced in March or April.

Dartmouth College is committed to the principle of equal opportunity for all its students, faculty, employees, and applicants for admission and employment. For that reason Dartmouth prohibits discrimination on the basis of race, color, religion, sex age, sexual orientation, national origin, disability, or status as a disabled or Vietnam era veteran in its programs, organizations, and conditions of employment and admission.



The graduate computing lab is spacious and quiet
(L-R: Erik Tou, Giulio Genovese).

General Information

Computing

Graduate students in mathematics have access to a large number of Macintosh and Windows computers as well as UNIX/Linux Workstations either in their offices or in an adjacent graduate lab. The entire campus is wired with Ethernet and blanketed with wireless internet access, providing students with access to mail, file, and print servers and an array of larger public computers for intense computation or graphics work.

Housing

College-owned housing available to graduate students includes apartments and duplexes in the Sachem Village complex about two miles from campus and some condominiums the college has leased. There are also shared apart-

ments for single graduate students within walking distance of the campus. Frequently graduate students will join together to rent a nearby home or apartment; the Graduate Student Council maintains housing listings as part of the Guide for Students in the Graduate Studies website (<http://www.dartmouth.edu/~gradstdy/>).

Location and Attractions

Dartmouth's location in Hanover, New Hampshire, a small picturesque town, offers exceptional opportunities for summer and winter outdoor recreation. The Hopkins Center for the Arts showcases theater, dance, and music performances as well as screening movies. The Hood Museum of Art has a permanent collection stretching back to 1772 and hosts a variety of traveling exhibits, generally two at a time. Hanover is easily accessible by plane (Manchester-Boston Regional Airport and Boston's Logan International Airport are both convenient), bus, train, and via interstate highway or skis.