

# Number Theory

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The candidate will be expected to have a command of two of the three branches of number theory (elementary, algebraic and analytic) and have an acquaintance with the basic concepts, techniques and major theorems of the third.

## **Elementary Number Theory:**

1. Unique factorization theory in  $\mathbb{Z}$
2. Congruences
3. Primitive roots
4. Quadratic reciprocity
5. Gauss sums
6. Number-theoretic functions and Mobius inversion
7. The distribution of primes
8. One of Diophantine equations or the theory of partitions

## **Analytic Number Theory:**

1. Dirichlet series and Euler products
2. Characters on abelian groups
3. The Riemann zeta function
  - (a) Poisson summation, analytic continuation, and the functional equation for the zeta function
  - (b) Zeros of the zeta function, trivial, non-trivial, Riemann Hypothesis
4.  $L$ -series, Prime Number Theorem, distribution of prime numbers
5. Dirichlet's theorem on primes in arithmetic progressions
6. Prime Number theorem for arithmetic progressions

# **Algebraic Number Theory:**

## **Global Theory:**

1. Dedekind domains
2. Ring of integers in a number field
3. Integral basis, fractional ideals, residue class field, norm of an ideal
4. Ideal class group and class number
5. Minkowski's theorem on convex regions
6. Dirichlet's Unit theorem
7. Decomposition of prime ideals in
  - (a) arbitrary extensions of number fields
  - (b) Galois extensions of number fields
  - (c) Abelian extensions of number fields
8. Ramification and inertial degrees, discriminant and different
9. Decomposition and Inertia groups and fields
10. Frobenius automorphism, Artin symbol
11. Kronecker-Weber theorem
12. Examples: Quadratic and Cyclotomic fields

## **Local Theory:**

1. Valuations
2. Approximation theorem for valuations
3. Completions of number fields
4. Local fields and Hensel's lemma
5. Examples:  $p$ -adic numbers

## References:

### Elementary Number Theory

1. Adams, Goldstein: *Introduction to Number Theory*
2. Apostol: *Introduction to Analytic Number Theory*
3. Ireland, Rosen: *A Classical Introduction to Modern Number Theory*
4. LeVeque: *Topics in Number Theory*
5. Niven, Zuckerman: *Number Theory*

### Analytic Number Theory

1. Apostol: *Introduction to Analytic Number Theory*
2. Chandrasekharan: *Introduction to Analytic Number Theory*
3. Rademacher: *Topics in Analytic Number Theory*
4. Titchmarsh: *Theory of the Riemann Zeta Function*

### Algebraic Number Theory

(G = Global; L = Local)

1. (L) Artin: *Algebraic Numbers and Algebraic Functions*
2. (G,L) Cassels, Frohlich: *Algebraic Number Theory*
3. (G,L) Golstein: *Analytic Number Theory (Chapters 1-6)*
4. (G,L) Janusz: *Algebraic Number Fields*
5. (G) Lang: *Algebraic Number Theory*
6. (G) Marcus: *Number Fields*
7. (G) Ribenboim *Algebraic Numbers*
8. (L) Weiss: *Algebraic Number Theory*