Series

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Definition of an infinite series

Given a sequence $\{a_n\}$, a **series (or infinite series)** is the addition of the terms of the sequence:

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$$

Definition of Sum, Convergence and Divergence

Given a series $\sum_{n=1}^{\infty} a_n$, let

$$s_n = a_1 + a_2 + \cdots + a_n$$

be the n-th **partial sum**.

If the sequence $\{s_n\}$ converges and $\lim_{n\to\infty} s_n = s$ is a real number, then the series $\sum_{n=1}^{\infty} a_n$ is called **convergent** and we write

$$a_1 + a_2 + \cdots = s \text{ or } \sum_{n=1}^{\infty} a_n = s$$

The number **s** is called the **sum** of the series.

Otherwise, we call the series **divergent**.

The Geometric Series

The **geometric series**

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \cdots$$

where a and r are constant real numbers. This series is convergent if $|r| < 1$ and its sum is

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$$

If $|r| \geq 1$, the geometric series diverges.

Test for Convergence

Theorem: If a series $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n \to \infty} a_n = 0$

The Test for Convergence:

If $\lim_{n \to \infty} a_n$ does not exist or if $\lim_{n \to \infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.