

# Alternating Series

January 22, 2007

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# The Alternating Series Test

- The  $n$ th term is of the form

$$a_n = (-1)^{n-1}b_n \text{ or } a_n = (-1)^nb_n,$$

where each  $b_n$  is a positive number.

- **The Alternating Series Test:** If the alternating series

$$\sum_{n=1}^{\infty} (-1)^n b_n$$

satisfies

$$b_{n+1} \leq b_n \text{ for all } n$$
$$\lim_{n \rightarrow \infty} b_n = 0$$

then the series is convergent.

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- $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+1}$

# Absolutely Convergent and Conditional Convergent Series

- A series  $\sum a_n$  is called **absolutely convergent** if the series of absolute values  $\sum |a_n|$  is convergent.
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- **Theorem:** If a series  $\sum a_n$  is absolutely convergent, then it is convergent.

# The Ratio Test

1. If

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1,$$

then the series  $\sum a_n$  is absolutely convergent.

2. If

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1 \quad \text{or} \quad \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty,$$

then the series  $\sum a_n$  is divergent.

3. If

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1,$$

the Ratio Test is inconclusive.

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- $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ .
- $\sum \frac{(n+3)!}{3!n!3^n}$