

Lagrange Multipliers

Lecture 28

February 7, 2007

Example

Find the extreme values of $f(x, y) = x^2 + 2y^2$ on the set

$$D = \{(x, y) : x^2 + y^2 \leq 1\}.$$

Fact

To find the maximum and minimum values of $f(x, y, z)$ subject to constraint $g(x, y, z) = k$:

Fact

To find the maximum and minimum values of $f(x, y, z)$ subject to constraint $g(x, y, z) = k$:

- 1 Find all values of x, y, z , and λ such that

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$$

and

$$g(x, y, z) = k$$

Fact

To find the maximum and minimum values of $f(x, y, z)$ subject to constraint $g(x, y, z) = k$:

- 1 Find all values of x, y, z , and λ such that

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$$

and

$$g(x, y, z) = k$$

- 2 Evaluate f at all the points (x, y, z) that result from step 1. The largest of these values is the maximum value of f ; the smallest is the minimum value of f .

Example

Example

- Let's use the Lagrange multiplier method to find the extreme values of $f(x, y) = x^2 + 2y^2$ on the unit circle.

Example

- Let's use the Lagrange multiplier method to find the extreme values of $f(x, y) = x^2 + 2y^2$ on the unit circle.
- Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3, 1, -1)$.

Thank you and good luck!
The End!