

Representations of Functions as Power Series

January 26, 2007

Lecture 11

Example

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- $\frac{x^4}{x+3}$

Differentiation and Integration of Power Series

If the power series $\sum c_n(x - a)^n$ has radius of convergence $R > 0$, then the function f defined by

$$f(x) = \sum_{n=0}^{\infty} c_n(x - a)^n$$

is differentiable (and continuous) on the interval $(a - R, a + R)$ and

1. $f'(x) = c_1 + 2c_2(x - a) + 3c_3(x - a)^2 + \dots$ (term by term differentiation)
2. $\int f(x)dx = C + c_0(x - a) + c_1\frac{(x-a)^2}{2} + c_2\frac{(x-a)^3}{3} + \dots$ (term by term integration)

The radius of convergence of the power series in these equations are both R .

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- Find a power series representation for $\ln(1+x)$ and its radius of convergence.
- Same question for $\tan^{-1} x$.
- $f(x) = \frac{x^2}{(1+x)^2}$.

Examples ...

Evaluate the indefinite integral as power series. What is the radius of convergence?

- $\int \frac{\ln(1-t)}{t} dt$

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- $\int \tan^{-1}(x^2) dx.$