Maxima and Minima

November 27

Lecture 28

Second Derivative Test

- Suppose the second partial derivatives of f are continuous on a disk with center (a, b), and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$. Let

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx}f_{yy} - (f_{xy})^2.$$

1. If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.

2. If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.

3. If $D < 0$, then $f(a, b)$ is not a local maximum or minimum. In this case the point (a, b) is called a **saddle point** of f.

Examples

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- Find three positive numbers x, y, and z whose sum is 100 and whose product is maximum.

- A rectangular box without a lid is to be made from 12 m^2 of cardboard. Find the maximum volume of such a box.

- A rectangular box without a lid is to be made from 12 m^2 of cardboard. Find the maximum volume of such a box.

- Find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in the ellipsoid

$$9x^2 + 36y^2 + 4z^2 = 36.$$

Absolute Maximum and Minimum Values

- A **closed set** in $\mathbb{R}^2$ is one that contains all its boundary points.
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- *Extreme Value Theorem for Functions of Two Variables* If f is continuous on a closed, bounded set D in $\mathbb{R}^2$, then f attains an absolute maximum value $f(x_1, y_1)$ and an absolute minimum value $f(x_2, y_2)$ at some points (x_1, y_1) and (x_2, y_2) in D.

Extension of the Closed Interval Method

- To find the absolute maximum and minimum values of a continuous function f on a closed, bounded set D:

  1. Find the values of f at the critical points of f in D.
  2. Find the extreme values of f on the boundary of D.
  3. The largest of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Examples

- Find the absolute maximum and minimum values of the function $f(x.y) = x^2 - 2xy + 2y$ on the rectangle

$$D = \{(x,y)|0 \leq x \leq 3\ ,\ 0 \leq y \leq 2\}.$$

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- Find the absolute maximum and minimum values of the function $f(x, y) = 3 + xy - x - 2y$ on the closed triangular region with vertices $(1, 0)$, $(5, 0)$, and $(1, 4)$.