Lines in $\mathbb{R}^3$

October 27, 2006

Lecture 17

The vector equation of a line

- A line L is determined when we know a point $P_0(x_0, y_0, z_0)$ on L and the direction of L.
- Let v be a vector parallel to L and let r_0 the position vector of P_0
- The **vector equation** of L is

$$\mathbf{r} = \mathbf{r_0} + t\mathbf{v},$$

where t is the **parameter.**

Examples

- Find the vector equation for the line trough the point $P(-1, 2, 2)$ and parallel to the vector $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

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The parametric equations of a line

- If $\mathbf{r} = \langle x, y, z \rangle$, $\mathbf{v} = \langle a, b, c \rangle$, and $\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$, then

$$\langle x, y, z \rangle = \langle x_0 + ta, y_0 + tb, z_0 + tc \rangle$$

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- The **parametric equations:**

$$\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}$$

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The symmetric equations of a line

- If $\mathbf{v} = \langle a, b, c \rangle$, then a, b, c are called **direction numbers**.

- If none of a, b, c is 0, the symmetric equation of the line is

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

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The line segment between two points

- The line segment from $\mathbf{r}_0$ and $\mathbf{r}_1$ is given by the vector equation

$$\mathbf{r}(t) = (1-t)\mathbf{r}_0 + t\mathbf{r}_1 \quad 0 \leq t \leq 1$$

Examples

- Find symmetric equations for the line trough $(4, 1, 0)$ that is parallel to the line with parametric equations $x = 1 + 2t, y = 2 + 3t, z = 1 - t$.

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- Find the point of intersection of this new line with each of the coordinate planes.

- Determine whether the lines

$$L_1 : \frac{x-3}{4} = \frac{y+7}{5} = \frac{z-3}{2}$$

and

$$L_2 : \frac{x+3}{2} = \frac{y-2}{2} = \frac{z+1}{5}$$

intersect, are skew, or are parallel.

- Determine whether the lines

$$L_1 : \frac{x+2}{3} = \frac{y-5}{7} = \frac{z-3}{4}$$

and

$$L_2 : \frac{x+1}{6} = \frac{y-2}{14} = \frac{z+3}{8}$$

intersect, are skew, or are parallel.