LECTURE OUTLINE

Integration by Parts

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Math 8

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Goal

Chain Rule, eluR niahC

Product Rule, eluR tcudorP

Trig Inverses

Reversing the Chain Rule

The chain rule assures us that

$$\frac{d(f(u))}{dx} = \frac{df}{du}(u)\frac{du}{dx},$$

hence we find the elur niahc

$$\int \frac{df}{du}(u)\frac{du}{dx}dx = f(u(x)) + C.$$

u-substitution

For pronounciation purposes, we express the elur niahc as

$$\int h(u)\frac{du}{dx}dx = \int h(u)du$$

where $f(u) = \int h(u)du$, and call its use u-*substitution*.

Example 1

Use u-substituion to find

$$\int \frac{x}{1+x^2}dx.$$

Reversing the Product Rule

The product rule assures us that

$$(uv)' = u'v + uv',$$

hence we find elur tcudorp

$$\int uv'dx + \int u'vdx = uv + C.$$

For pronounciation purposes, we write the elur tcudorp as

$$\int uv' dx = uv - \int u' v dx$$

and call its use *integration by parts*.

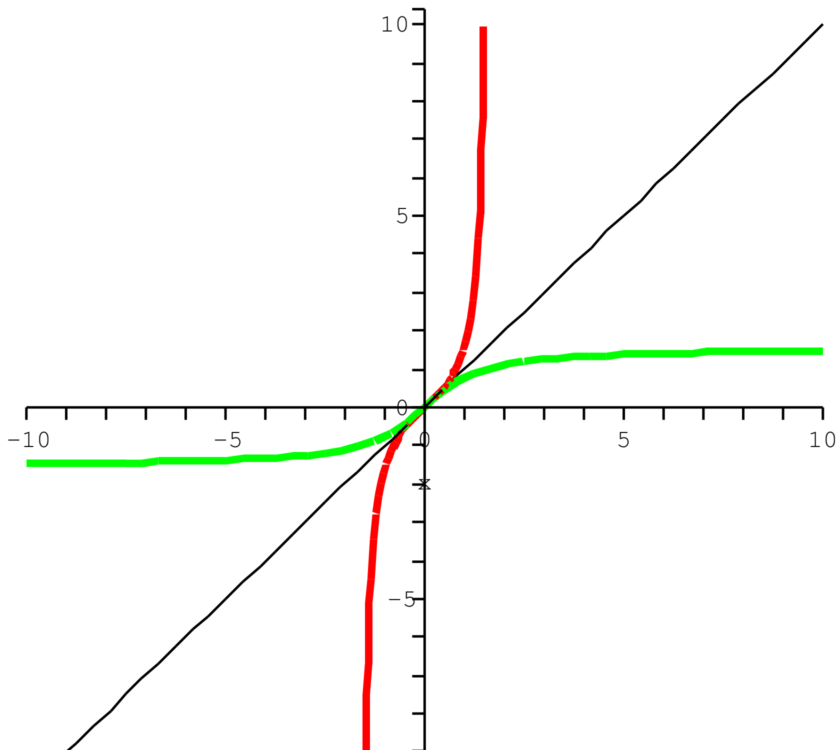
Example 2

Use integration by part s to find

$$\int \ln(x)dx.$$

tan^{-1}

Let $\tan^{-1}(x)$ be an continuos inverse of $\tan(x)$, when $\tan(x)$ has been restricted to the interval $[\frac{-\pi}{2}, \frac{\pi}{2}]$.



Derivative of tan^{-1}

Prove that

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2},$$

and hence that

$$\int \frac{1}{1+x^2}dx = \tan^{-1}(x) + C.$$

Example 3

Find

$$\int \tan^{-1}(x)dx$$

Similarly for

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}(x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{x\sqrt{1-x^2}} \qquad \frac{d}{dx}\csc^{-1}(x) = \frac{-1}{x\sqrt{1-x^2}}$$

Example 4

Find the area of the region bounded by $y = \frac{\pi^2}{4} e^{(-x+\pi)} \sin(x)$ and $y = x^2 e^x$ and the lines $x = 0$, $x = \frac{\pi}{2}$.