

# Homework due 27/10

$$\begin{aligned} \underline{1.} \quad (a.) \quad f(x) &= x^{-2} & f(1) &= 1 \\ f'(x) &= -2x^{-3} & f'(1) &= -2 \\ f''(x) &= 6x^{-4} & f''(1) &= 6 \end{aligned}$$

so the 2<sup>nd</sup>-degree Taylor approximation to  $f$  at 1 is

$$T_2(x) = 1 + (-2)(x-1) + \frac{6}{2!}(x-1)^2 = 1 - 2(x-1) + 3(x-1)^2$$

$$(b.) \quad f'''(x) = -24x^{-5}. \quad \text{On } (0.9, 1.1) \quad x^{-5} > 0, \text{ so}$$

$$|f'''(x)| = 24x^{-5}.$$

This is decreasing, so for  $x$  in  $(0.9, 1.1)$ ,

$$|f'''(x)| \leq |f'''(0.9)| = 24 \cdot 0.9^{-5}$$

$$\text{So take } M = 24 \cdot 0.9^{-5}$$

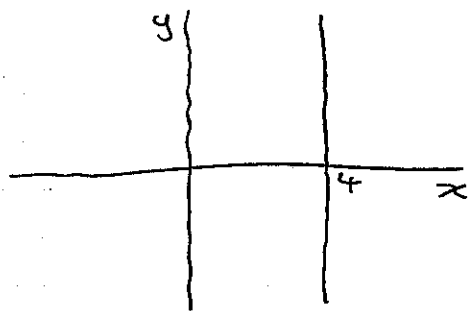
Thus by Taylor's Inequality,

$$|R_2(x)| \leq \frac{24 \cdot 0.9^{-5}}{3!} |x-1|^3.$$

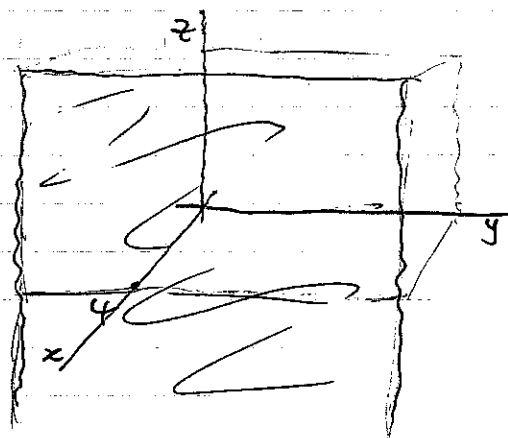
$\max |x-1| = 0.1$ , so for all  $x$  in  $(0.9, 1.1)$

$$|R_2(x)| \leq \frac{24}{6} \cdot 0.9^{-5} \cdot 0.1^3 \approx 0.00677$$

2. (a)  $x=4$  is a line in  $\mathbb{R}^2$ :



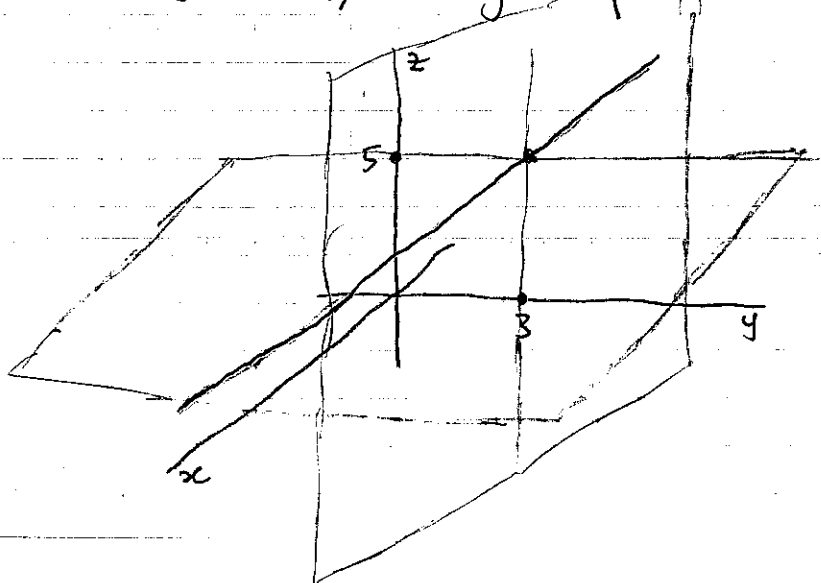
but a plane in  $\mathbb{R}^3$ :



(b)  $y=3$  is the plane of points parallel to the  $(x,z)$ -plane through the point  $(0,0,3)$ .

$z=5$  is the plane of points  $(x,y,5)$  for arbitrary  $x,y$ .

The points satisfying  $y=3$  and  $z=5$  are a line parallel to the  $x$ -axis, namely all points  $(x,3,5)$



3.  $|PQ|^2 = \sqrt{(1-(-2))^2 + (2-4)^2 + (-1+0)^2} = \sqrt{9+4+1} = \sqrt{14}$

$$|PR| = \sqrt{(-1-2)^2 + (1-4)^2 + (2-0)^2} = \sqrt{9+9+4} = \sqrt{22}$$

$$|QR| = \sqrt{(-1-0)^2 + (1-2)^2 + (2-(-1))^2} = \sqrt{4+1+9} = \sqrt{14}$$

So the triangle is equilateral.

4. (a)  ~~$\vec{B}-\vec{A} = (7-5, 9-1, -1-3) = (2, 8, -4)$~~

~~$$\vec{C}-\vec{A} = (1-5, -15-1, 11-3) = (-4, -16, 8)$$~~

$$\vec{B}-\vec{A} = (7-5, 9-1, -1-3) = (2, 8, -4)$$

$$\vec{C}-\vec{A} = (1-5, -15-1, 11-3) = (-4, -16, 8) = -2 \cdot (2, 8, -4).$$

Since B and C both lie in the direction of the vector  $(2, 8, -4)$  from A, A, B, C lie on a straight line.

(b)  $\vec{L}-\vec{K} = (1-0, 2-3, -2-(-4)) = (1, -1, 2)$

$$\vec{M}-\vec{K} = (3-0, 0-3, 1-(-4)) = (3, -3, 5) = 3 \cdot (1, -1, \frac{5}{3})$$

So M and L lie in different directions from K, thus these three do not lie on a straight line.

5.  $(x-1)^2 + (y+4)^2 + (z-3)^2 = 5^2$

The intersection with the  $(x, z)$ -plane is the set of points which also satisfy  $y=0$ , i.e.

$$(x-1)^2 + 4^2 + (z-3)^2 = 25$$

$$(x-1)^2 + (z-3)^2 = 25 - 4^2 = 9$$

This is a circle, centre  $(1, 0, 3)$ , radius 3.

6.

$$x^2 + y^2 + z^2 = 4x - 2y$$

$$x^2 - 4x + y^2 + 2y + z^2 = 0$$

$$(x-2)^2 - 4 + (y+1)^2 - 1 + z^2 = 0$$

$$(x-2)^2 + (y+1)^2 + z^2 = 1+4=5.$$

This has centre  $(2, -1, 0)$  a radius  $\sqrt{5}$ .

~~7.~~

7.

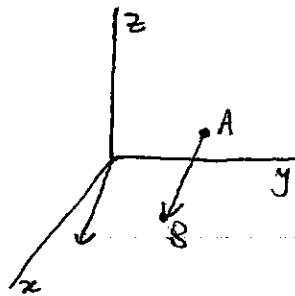
The outer bound is the sphere of radius 5

and the inner bound is the sphere of radius 1

so the region  $1 \leq x^2 + y^2 + z^2 \leq 25$  are those points between these two spheres.

8.

$$\underline{a} = (2-0, 3-3, -1-1) = (2, 0, -2)$$



9.

$$|\underline{a}| = \sqrt{(-3)^2 + (-4)^2 + (-1)^2} = \sqrt{26}$$

$$\underline{a} + \underline{b} = (-3+6, -4+2, -1-3) = (3, -2, -4)$$

$$\underline{a} - \underline{b} = (-3-6, -4-2, -1-(-3)) = (-9, -6, 2)$$

$$2\underline{a} = (2 \cdot (-3), 2 \cdot (-4), 2 \cdot (-1)) = (-6, -8, -2)$$

$$3\underline{a} + 4\underline{b} = (3 \cdot (-3) + 4 \cdot 6, 3 \cdot (-4) + 4 \cdot 2, 3 \cdot (-1) + 4 \cdot (-3)) = (15, -4, -15)$$

10.  $|\underline{a}| = \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6}$

$$\underline{a} + \underline{b} = \underline{i} + (-2+1)\underline{j} + \underline{k} + 2\underline{k} = \underline{i} - \underline{j} + 3\underline{k}$$

$$\underline{a} - \underline{b} = \underline{i} + (-2-1)\underline{j} + (1-2)\underline{k} = \underline{i} - 3\underline{j} - \underline{k}$$

$$2\underline{a} = 2\underline{i} + 2(-2)\underline{j} + 2\underline{k} = 2\underline{i} - 4\underline{j} + 2\underline{k}$$

$$3\underline{a} + 4\underline{b} = 3\underline{i} + (3(-2)+4)\underline{j} + (3+4(2))\underline{k} = 3\underline{i} - 2\underline{j} + 15\underline{k}$$

11.  $|\langle 9, -5 \rangle| = \sqrt{9^2 + (-5)^2} = \sqrt{106}$

so  $\frac{1}{\sqrt{106}} \cdot \langle 9, -5 \rangle = \langle \frac{9}{\sqrt{106}}, -\frac{5}{\sqrt{106}} \rangle$  is a unit vector  
in the same direction as  $\langle 9, -5 \rangle$ .

12.  $|\underline{8i} - \underline{j} + 4\underline{k}| = \sqrt{8^2 + (-1)^2 + 4^2} = \sqrt{81} = 9$

so the unit vector in this direction is  $\frac{8}{9}\underline{i} - \frac{1}{9}\underline{j} + \frac{4}{9}\underline{k}$ .

13.  $|\langle -2, 4, 2 \rangle| = \sqrt{(-2)^2 + 4^2 + 2^2} = \sqrt{24} = \sqrt{4 \cdot 6} = 2\sqrt{6}$

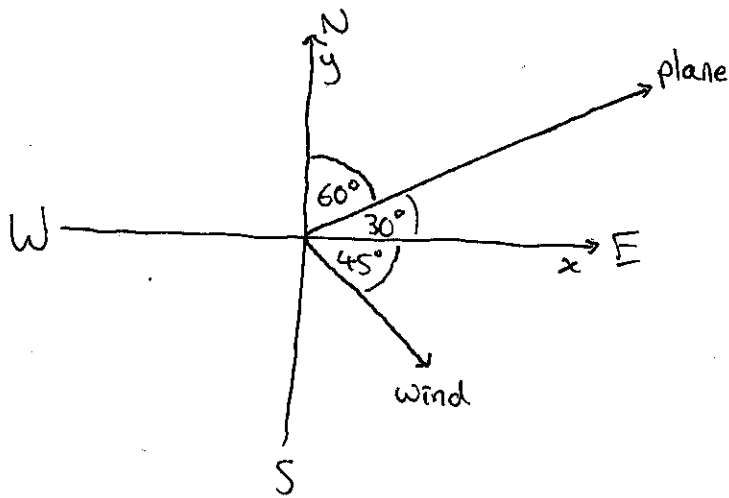
so the unit vector in this direction is  $\langle \frac{-2}{2\sqrt{6}}, \frac{4}{2\sqrt{6}}, \frac{2}{2\sqrt{6}} \rangle$   
 $= \langle -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle$ .

Thus the vector of length 6 is  $\langle \frac{-6}{\sqrt{6}}, \frac{6 \cdot 2}{\sqrt{6}}, \frac{6}{\sqrt{6}} \rangle = \langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6} \rangle$

as can be seen by checking:

$$|\langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6} \rangle| = \sqrt{(-\sqrt{6})^2 + (2\sqrt{6})^2 + (\sqrt{6})^2} = \sqrt{6 + 4 \cdot 6 + 6} = \sqrt{36} = 6.$$

14.



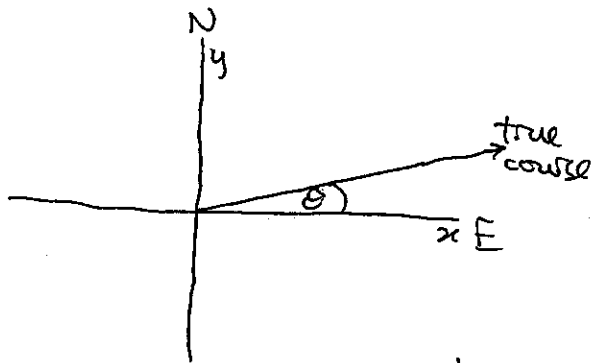
$$\underline{v}_{\text{wind}} = 50(\cos 45^\circ \underline{i} - \sin 45^\circ \underline{j}) = \frac{50}{\sqrt{2}} \underline{i} - \frac{50}{\sqrt{2}} \underline{j}$$

$$\underline{v}_{\text{plane}} = 250(\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j}) = \frac{250\sqrt{3}}{2} \underline{i} + \frac{250}{2} \underline{j} = 125\sqrt{3} \underline{i} + 125 \underline{j}$$

so the velocity relative to the ground is

$$\underline{v} = \underline{v}_{\text{wind}} + \underline{v}_{\text{plane}} = \left( \frac{50}{\sqrt{2}} + 125\sqrt{3} \right) \underline{i} + \left( 125 - \frac{50}{\sqrt{2}} \right) \underline{j}$$

The ground speed is  $|\underline{v}| = \sqrt{\left( \frac{50}{\sqrt{2}} + 125\sqrt{3} \right)^2 + \left( 125 - \frac{50}{\sqrt{2}} \right)^2} \approx 267 \text{ kmh}^{-1}$



$$\theta = \tan^{-1} \left( \frac{125 - \frac{50}{\sqrt{2}}}{\frac{50}{\sqrt{2}} + 125\sqrt{3}} \right) \approx 20^\circ$$

so the course of the plane is  $N 70^\circ E$