

**Math 71**  
Homework 6

- page 122: 8, 10, 14
- pp 165-166: 2(b,e), 3(b,e)
- The following additional problems:
  1. Let  $G$  be a group of order 48. Show that  $G$  has a normal subgroup of order 8 or 16. Some hints are provided.
    - (a) If there is more than one Sylow 2-subgroup, let  $H$  and  $K$  be any two (distinct) Sylow 2-subgroups. Show that  $|H \cap K| = 8$ .
    - (b) Show that for the  $H$  and  $K$  above, that  $H, K \subseteq N_G(H \cap K)$ .
    - (c) Show that  $G = N_G(H \cap K)$ .
  2. Let  $G$  be a group of order  $231 = 3 \cdot 7 \cdot 11$ , and suppose that there is only one Sylow 3-subgroup. Show that  $G$  is cyclic.