Probability From Fair Price

Math 5 Crew

Department of Mathematics

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- In the process, we learned the FFMP

$$E(cX + dY) = cE(X) + dE(Y),$$

- and (provided X and Y are independent!) the SFMP

$$E(XY) = E(X)E(Y)$$

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- then we say the *Probability* that U occurs is $E(Z)$ and use the notation $P(E) = E(Z)$.
- Notice, from this view $P($Bush is the next president$) = E(X/100) = 0.6465$, while the $P($Lord of the Rings wins Best Picture$) = 0.8385$.

Discussion Question

- Let Z be the bet which is one if at least one pair of you mothers share a birthday (month and day) and zero otherwise. For what price would you be willing to sell Z and for what price would you be willing to buy Z?

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- What do you feel would be Z's Fair Price in an efficient market?
- How about the bet W that at least 2 pairs of your mothers share the same birthday?

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- We need to find a bet which is 1 if they both win and zero otherwise. Notice, $Z = \frac{X}{100}\frac{Y}{100}$ has this property.
- Hence using the FFMP and SFMP $P(E)$ equals

$$E(Z) = E(\frac{X}{100}\frac{Y}{100}) = \frac{1}{10000}E(XY) = \frac{5420.90}{10000} = 0.542.$$

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$$P(U) = (0.6465)(0.8385) = 0.5420$$

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- Hence using the first and second fundamental mysteries $P(V)$ equals

$$E(Z) = E(\frac{X}{100} + \frac{Y}{100} - \frac{X}{100}\frac{Y}{100})$$

$$= \frac{E(X)}{100} + \frac{E(Y)}{100} - \frac{E(XY)}{10000}$$

$$= 0.6465 + 0.8385 - 0.5420 = 0.9430$$

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- For example, the probability that George Bush fails to be the next president is

$$P(U^c) = 1 - P(U) = 1 - 0.6465 = 0.3535$$

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- We will now use this fact to analyze our Birthday Bet's Fair Price.