

MATH 46 WORKSHEET : regular perturbation

4/6/07
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Substitute $y(t) = y_0(t) + \varepsilon y_1(t) + \varepsilon^2 y_2(t) \dots$

into $y' = -y + \varepsilon y^2$ with IC $y(0) = 1$

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Collect ε^0 terms :

what IC does $y_0(t)$ satisfy? solve for it.

Collect ε^1 terms :

What IC does $y_1(t)$ satisfy? [Hint sub series into original IC] solve for it.

SOLUTIONS

see p. 88-90.

Substitute $y(t) = y_0(t) + \epsilon y_1(t) + \epsilon^2 y_2(t) \dots$

into $y' = -y + \epsilon y^2$ with IC $y(0) = 1$

$$y'_0 + \epsilon y'_1 + \epsilon^2 y'_2 + \dots = \underbrace{-y_0 - \epsilon y_1 - \epsilon^2 y_2 + \dots}_{\text{from } -y} + \underbrace{\epsilon y_0^2 + 2\epsilon^2 y_0 y_1 + O(\epsilon^3)}_{\text{from } \epsilon y^2} + \dots$$

IC gives $y_0(0) + \epsilon y_1(0) + \epsilon^2 y_2(0) + \dots = 1$

Collect ϵ^0 terms: $y'_0 = -y_0$ ie $y'_0 + y_0 = 0$
ie $y_0(t) = A e^{-t}$

what IC does $y_0(t)$ satisfy? solve for it.

Taking ϵ^0 term in IC gives $y_0(0) = 1$ so $A = 1$

ie $y_0(t) = e^{-t}$

Collect ϵ^1 terms: $y'_1 = -y_1 + y_0^2$

What IC does $y_1(t)$ satisfy? [Hint sub series into original IC] solve for it.

$y_1(0) = 0$

so $y'_1 + y_1 = y_0^2 = e^{-2t}$ integrating factor e^t
 $e^t y_1 = \int e^{-t} dt + c$ IC makes $c = +1$
 $y_1 = e^{-t}(e^{-t} + c) = -e^{-2t} + c e^{-t} = -e^{-2t} + e^{-t}$