

Show that the transformation  $w = u^{1-n}$  makes the 'Bernoulli eqn'

$$u' + p(t)u = q(t)u^n$$

← which looks nonlinear

into a linear eqn. What are the new  $\tilde{p}(t)$  and  $\tilde{q}(t)$ ? in the linear eqn?

What method(s) would you use on following? you may need one followed by another.

i)  $u'' + 2t(u')^2 = 0$

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iii)  $u'' = 2u + (u)^3$

iv)  $u'' + u' = u + \ln t$

v)  $\frac{u'}{u} = t^2 u^3 + \frac{1}{t}$

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$w = u^{1-n}$

invert to get  $u = w^{\frac{1}{1-n}}$   
 so  $\frac{du}{dt} = \frac{du}{dw} \frac{dw}{dt} = \frac{w^{\frac{n}{1-n}}}{1-n} w'$  ← (This gave people trouble! Please get back into your algebra)

in ODE:  $\frac{w^{\frac{n}{1-n}}}{1-n} w' + p(t) w^{\frac{1}{1-n}} = q(t) (w^{\frac{1}{1-n}})^n = q(t) w^{\frac{n}{1-n}}$

lt. by  $\frac{1-n}{w^{\frac{n}{1-n}}}$ :  $w' + \underbrace{(1-n)p(t)}_{\text{the 'p(t)'}} w = \underbrace{(1-n)q(t)}_{\text{the 'q(t)'}}$

What method(s) would you use on following? you may need one followed by another.

i)  $u'' + 2t(u')^2 = 0$   $v = u'$  then separate variables (1st order)

ii)  $u'' + 3u' + 2u = t$  Und. Coeffs.

iii)  $u'' = 2u + (u')^3$   $v = u'$  then  $G(u, u', u'') = 0$  ( indep. of  $t$ , use  $G(u, v, v \frac{dv}{du}) = 0$  to get  $v(u)$  then get  $u(t)$  by 1st-order ODE

iv)  $u'' + u' = u + \ln t$  Variation of Parameters.

v)  $\frac{u'}{u} = t^2 u^3 + \frac{1}{t}$  mult. by  $u$ , then it's a Bernoulli Eqn. as above.

more elegantly, implicit differentiate to get  $\frac{dw}{dt} = (1-n)u'u^{-n}$  (\*)

divide ODE by  $u^n$ :  $\frac{u'u^{-n}}{\frac{1}{1-n} w' \text{ by (*)}} + p(t) \frac{u^{1-n}}{w} = q(t)$

same result.