MATH 46 WORKSHEET : Converting IVP $\to$ Volterra. 5/1/08 Barnett

using the Lemma : $\int_0^x \int_0^y u(r)\,dr\,dy = \int_0^x (x-y)u(y)\,dy$

- Convert the IVP : $\begin{cases} u'' + q(t)u = g(t) \\ u(0) = A \\ u'(0) = B \end{cases}$

into a Volterra equation of the form $\underbrace{K}_{\text{integral operator}} u - \lambda u = f$

[Hint: integrate $\int_0^t ds$ twice:]

- If time, now try on $u'' + p(t)u' + q(t)u = g(t)$ [see p. 233]

[Hint: use by parts after first $\int_0^t ds$]

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— SOLUTIONS —

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- Convert the IVP : $\begin{cases} u'' + q(t)u = g(t) \\ u(0) = A \\ u'(0) = B \end{cases}$

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[Hint: integrate $\int_0^t ds$ twice:]

$$u''(s) + q(s)\,u(s) = g(s)$$

$\int_0^t ds$ ↓ Fund. Thm. Calculus.

$$u'(t) - B + \int_0^t q(s)u(s)\,ds = \int_0^t g(s)\,ds$$

$\int_0^t dr$ ↓ change var to r from t.

use as "u" in Lemma.

$$u(t) - Bt - A + \int_0^t\int_0^r \boxed{q(s)u(s)}\,ds\,dr + \int_0^t\int_0^r g(s)\,ds\,dr$$

$\lambda = -1$.

$$u(t) + \int_0^t \overbrace{(t-s)\,q(s)}^{k(t,s)\text{ kernel}}\,u(s)\,ds = \underbrace{\int_0^t (t-s)g(s)\,ds + A + Bt}_{f(t)}$$

- If time, now try on $u'' + \underbrace{p(t)\,u'} + q(t)\,u = g(t)$

[Hint: use by parts after first $\int_0^t ds$]

$\int_0^t ds$ ↙ Just do this term since all others are done above [see p. 233]

$$\underbrace{\int_0^t p(s)\,u'(s)\,ds}_{\text{by parts}}$$

$$-\int_0^t p'(s)\,u(s)\,ds + \left[p(s)\,u'(s)\right]_0^t \to p(t)\,u'(t) - Bp(0)$$

$\int_0^t dr$

first 3 terms all contribute to kernel.

$$\text{Lemma} \quad -\int_0^t\int_0^r p'(s)u(s)\,ds + \underbrace{\int_0^t p(r)\,u'(r)\,dr}_{\text{by parts}} - Bp(0)t$$

$$-\int_0^t (t-s)\,p'(s)\,u(s)\,ds - \int_0^t p'(s)u(s)\,ds + p(t)u(t) - p(0)A$$

See p. 233-4.