MATH 46 WORKSHEET : Green's Functions

5/12/07
Barnett

Consider $A = -\frac{d^2}{dx^2}$ on $[0,1]$ with Dirichlet BCs

We wish to find the Green's func. to solve $Au = f$ with $u(0)=0$, $u(1)=0$

Write general solution to $Au = 0$

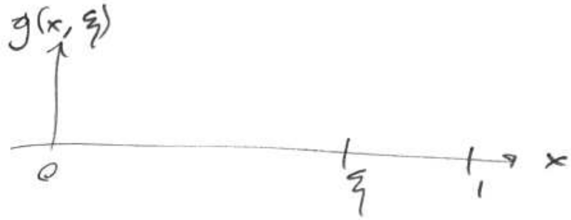
Solve for $u_1(x)$ which obeys only left-end BC :

$u_2(x)$ " " " right-end BC :

Compute Wronskian $W(x)$:

Write $g(x,\xi)$:

Sketch it for fixed ξ:

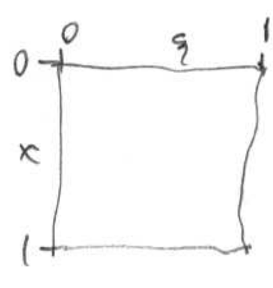


(is it continuous everywhere?)

(what is the jump in gradients?)

Write $g(\xi,x)$ -- notice anything?

Sketch $g(x,\xi)$ in the plane:



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$\swarrow p \equiv 1, \; q \equiv 0.$

Consider $A = -\frac{d^2}{dx^2}$ on $[0,1]$ with Dirichlet BCs

We wish to find the Green's func. to solve $Au = f$ with $u(0)=0$, $u(1)=0$

Write general solution to $Au = 0$

$u'' = 0$
so $u(x) = Ax + B$.

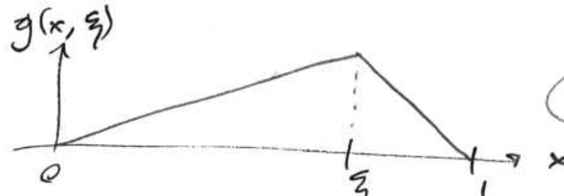
Solve for $u_1(x)$ which obeys only left-end BC : x

$u_2(x)$ " " " right-end BC : $1-x$

Compute Wronskian $W(x)$: $u_1u_2' - u_1'u_2 = x - (1-x)$
$= -1$

Write $g(x,\xi) = -\frac{1}{p(\xi)W(\xi)} \begin{cases} u_1(x)\,u_2(\xi), & x<\xi \\ u_2(x)\,u_1(\xi), & x>\xi \end{cases} = \begin{cases} x(1-\xi) & x<\xi \\ \xi(1-x) & x>\xi \end{cases}$

Sketch it for fixed ξ:



-1

(is it continuous everywhere? yes)

(What is the jump in gradient?)

Write $g(\xi, x)$ -- notice anything?
yes, $g(\xi, x) = g(x, \xi)$
symmetric.

Sketch $g(x,\xi)$ in the plane: