MATH 46 WORKSHEET : Dominant balancing

4/13/09
Barnett

B) Find the scaling of x with ε that makes two terms of equal order and others of lower order, in:

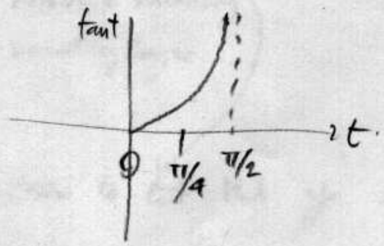
$$\varepsilon x^4 + \varepsilon x^3 - x^2 + 2x - 1 = 0$$

Find the leading-order term in each of the four roots:

If time, continue to higher corrections for these roots:

MATH 46 WORKSHEET : Asymptotic analysis, Dominant balancing. 4/11/07 Barnett

A) Is $f(t,\varepsilon) = \varepsilon \tan t$ *uniformly* convergent to zero on $(0, \pi/4)$?



yes since $|\tan t| \le$ const on $(0, \pi/4)$.

$(0, \pi/2)$? no since $\tan t$ unbounded on $(0, \pi/2)$

does $\varepsilon \tan t$ converge *pointwise* on $(0, \pi/2)$?

yes since for any $t \in (0, \pi/2)$, $\tan t$ is some number C and $f(t,\varepsilon) = C\varepsilon \to 0$ as $\varepsilon \to 0$.

B) Find the scaling of x with ε that makes two terms of equal order and others of lower order in

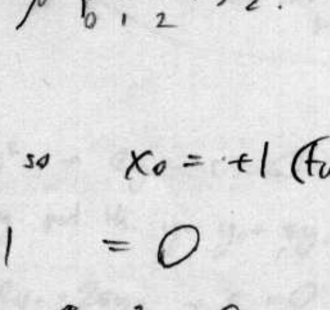
$$\varepsilon x^4 + \varepsilon x^3 - x^2 + 2x - 1 = 0$$

guess $x = \varepsilon^{-1}$? $\quad \varepsilon^{-3} \quad \varepsilon^{-2} \quad \varepsilon^{-2} \quad \varepsilon^{-1} \quad \varepsilon^{0}$

dominant

$x = \varepsilon^{-1/2}$? $\quad \varepsilon^{-1} \quad \varepsilon^{-1/2} \quad \varepsilon^{-1} \quad \varepsilon^{-1/2} \quad \varepsilon^{0}$

dominantly balanced



so $x = \varepsilon^{-1/2}$

Graphical way to solve: find the line connecting 2 points in the (power of ε, power of x) plane with all other points to the right (higher ε powers).

Find the *leading-order* term in each of the four roots:

regular roots (drop 1st two terms) $-x_0^2 + 2x_0 - 1 = 0$ so $x_0 = +1$ (twice)

sub. $x = \frac{y}{\varepsilon^{1/2}} \Rightarrow \varepsilon \frac{y^4}{\varepsilon^2} + \varepsilon \frac{y^3}{\varepsilon^{3/2}} - \frac{y^2}{\varepsilon} + 2\frac{y}{\varepsilon^{1/2}} - 1 = 0$

$\Rightarrow y^4 + \varepsilon^{1/2} y^3 - y^2 + 2\varepsilon^{1/2} y - \varepsilon = 0 \qquad y_0^4 - y_0^2 = 0$ want

ie $y_0 = 0$ twice, ± 1

If time, continue to higher corrections for these roots:

tricky: we'll do more later

$x = +1 + \frac{1}{4}\varepsilon^{1/2} + O(\varepsilon)$ (twice), $\pm \varepsilon^{-1/2} - \frac{3}{2} + O(\varepsilon^{1/2}) \ldots \qquad x = \pm \varepsilon^{-1/2}$