

MATH 46 WORKSHEET : Asymptotic analysis, Dominant balancing.

2/1/07
Borutt

A) Is $f(t, \varepsilon) = \varepsilon \tan t$ uniformly convergent to zero on $(0, \pi/4)$?

$(0, \pi/2)$?

does $\varepsilon \tan t$ converge pointwise on $(0, \pi/2)$?

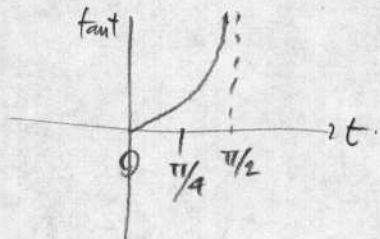
B) Find the scaling of x with ε that makes two terms of equal order and others of lower order in

$$\varepsilon x^4 + \varepsilon x^3 - x^2 + 2x - 1 = 0$$

Find the leading-order term in each of the four roots:

If time, continue to higher corrections for these roots:

A) Is $f(t, \epsilon) = \epsilon \tan t$ uniformly convergent to zero on $(0, \pi/4)$?



yes since $|\tan t| \leq \text{const}$ on $(0, \pi/4)$.

$(0, \pi/2)$?
 no since $\tan t$ unbounded on $(0, \pi/2)$

does $\epsilon \tan t$ converge pointwise on $(0, \pi/2)$?

yes since for any $t \in (0, \pi/2)$, $\tan t$ is some number C and $f(t, \epsilon) = C\epsilon \rightarrow 0$ as $\epsilon \rightarrow 0$.

B) Find the scaling of x with ϵ that makes two terms of equal order and others of lower order in

$$\epsilon x^4 + \epsilon x^3 - x^2 + 2x - 1 = 0$$

guess $x = \epsilon^{-1}$?

ϵ^{-3}

dominant

ϵ^{-2}

ϵ^{-2}

ϵ^{-1}

ϵ^0

$x = \epsilon^{-1/2}$?

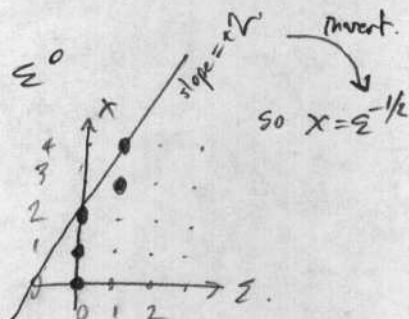
ϵ^{-1}

$\epsilon^{-1/2}$

ϵ^{-1}

$\epsilon^{-1/2}$

dominantly balanced



Graphical way to solve: find the line connecting 2 points in the (power of ϵ , power of x) plane with all other points to the right (higher ϵ powers).

Find the leading-order term in each of the four roots:

regular roots (drop 1st two terms) $-x^2 + 2x - 1 = 0$ so $x_0 = \pm 1$ (twice)

sub. $x = \frac{y}{\epsilon^{1/2}} \Rightarrow \epsilon \frac{y^4}{\epsilon^2} + \epsilon \frac{y^3}{\epsilon^{3/2}} - \frac{y^2}{\epsilon} + 2 \frac{y}{\epsilon^{1/2}} - 1 = 0$

$\Rightarrow y^4 + \epsilon^{1/2} y^3 - y^2 + 2\epsilon^{1/2} y - \epsilon = 0$

$y_0^4 - y_0^2 = 0$ want

ie $y_0 = 0$ twice, ± 1

If time, continue to higher corrections for these roots:
bricky: will do more later

$x = +1 + \frac{1}{4}\epsilon^{1/2} + O(\epsilon)$ (twice), $\pm \epsilon^{-1/2} - \frac{3}{2} + O(\epsilon^{1/2})$ $\times = \pm \epsilon^{-1/2}$