

# Math 46: X hour of 5/10/07

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We used Section 4.3.3, particularly Thms 4.12 and 4.13, to determine if the following had a solution, and then solve them. We made use of (4.31) a lot to get  $u(x)$  once the  $\mathbf{c}$  vector was found.

Let  $K$  operator have kernel  $k(x, y) = \sin x \sin y$ .

Then  $A$  is 1-by-1 matrix with entry  $\pi/2$ . Spectrum of  $K$  is then  $\pi/2$  (multiplicity 1, eigenfunction  $\sin x$ ), and 0 (infinite multiplicity, eigenspace all functions orthog to  $\{\beta_j\}$  that is  $\sin x$ )

Solve the following:

1.  $Ku - u = \sin 2x$

2.  $Ku - u = x$

(We used Maple to get the Fourier coefficient)

3.  $Ku - 3 \sin 2x$

4.  $Ku = 3 \sin x$ .

Answer key:

1.  $c_1 = 0$  so  $u = -\sin 2x$

2.  $c_1 = \frac{\pi}{1-\pi/2}$  so  $u = \frac{\pi}{1-\pi/2} \sin x - x$

3. no solution

4.  $u = \frac{6}{\pi} \sin x +$  (any function orthogonal to  $\sin x$ ). Infinitely-nonunique solution.