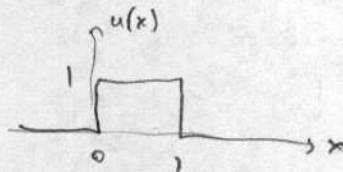


MATH 46 WORKSHEET : Convolution

5/25/07
Barnett

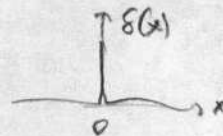
Work out the convolution of $u(x)$ and $v(x)$:

A)
$$u(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$



$v(x) = \text{same}$

B) $u(x) = \text{anything}$ B
 $v(x) = \delta(x)$ delta 'function' (distribution)



C) $u(x) = v(x) = e^{-\frac{x^2}{2}}$ Gaussian.

[Hint : complete the square then use Gaussian integral]

How wide is the answer compared to original Gaussian?

MATH 46 WORKSHEET : Convolution

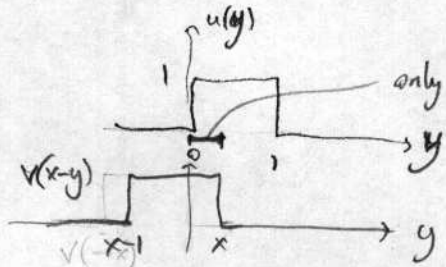
5/29/07
Barnett

Work out the convolution of $u(x)$ and $v(x)$:

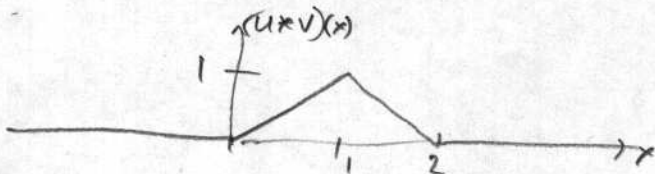
see "The Joy of Convolution" web applet to visualize better.

A) $u(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

$v(x) = \text{same}$

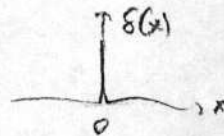


$$(u * v)(x) = \int_{-\infty}^{\infty} v(x-y) u(y) dy = \begin{cases} 0 & , x < 0 \text{ or } x > 2 \\ x & , 0 < x < 1 \\ 2-x & , 1 < x < 2 \end{cases}$$



B) $u(x) = \text{anything}$

$v(x) = \delta(x)$ delta 'function' (distribution)



$$(u * v)(x) = \int_{-\infty}^{\infty} \delta(x-y) u(y) dy = u(x) \text{ by 'sifting property' of } \delta.$$

get the same function back
(convolution w/ δ has no effect).

C) $u(x) = v(x) = e^{-x^2/2}$ Gaussian.

[Hint : complete the square then use Gaussian integral]

$$(u * v)(x) = \int_{-\infty}^{\infty} e^{-\frac{(x-y)^2}{2}} e^{-y^2/2} dy = \int e^{-\frac{x^2}{2} + xy - \frac{y^2}{2} - \frac{y^2}{2}} dy$$

$$= \int e^{-(y - x/2)^2 - x^2/4} dy = e^{-x^2/4} \int e^{-y^2} dy = \sqrt{\pi} e^{-x^2/4}$$

Gaussian with twice the variance

its width is $\sqrt{2}$ times as big ← How wide is the answer compared to original Gaussian?