

A) Solve for  $u(t)$  in the equation for stock decay with kernel  $k(t) = e^{-bt}$ :

$$ak(t) + \int_0^t k(t-\tau) u(\tau) d\tau = a$$

B) Differentiate to solve the following integral eqn (assume  $f'$  exists,  $a \neq 0$ ):

$$\int_0^t y u(y) dy - a u(t) = f(t) \quad \text{on } 0 \leq t \leq 1.$$

[convert to an ODE]

## SOLUTIONS

A) Solve for  $u(t)$  in the equation for steady decay with kernel  $k(t) = e^{-bt}$ :

$$ak(t) + \int_0^t k(t-\tau) u(\tau) d\tau = a$$

$$ae^{-bt} + \int_0^t e^{-b(t-\tau)} u(\tau) d\tau = a$$

$$e^{-bt} \int_0^t e^{b\tau} u(\tau) d\tau$$

Mult. by  $e^{bt}$ :

$$a + \int_0^t e^{b\tau} u(\tau) d\tau = ae^{bt}$$

$\frac{d}{dt}$

$$e^{bt} u(t) = ba e^{bt}$$

$$\Rightarrow \boxed{u(t) = ab}$$

B) Differentiate to solve the following integral eqn (assume  $f'$  exists,  $a \neq 0$ ):

$$\int_0^t y u(y) dy - a u(t) = f(t) \quad \text{on } 0 \leq t \leq 1.$$

[convert to an ODE]

$\frac{d}{dt}$

$$t u(t) - a u'(t) = f'(t)$$

$$\text{so } u'(t) - \frac{t}{a} u(t) = -\frac{f'(t)}{a}$$

$p(t)$

$q(t)$

is linear 1st order ODE

$$\text{Integrating factor is } e^{\int p dt} = e^{-\frac{1}{2a} t^2}$$

with IC:  $u(0) = \frac{f(0)}{a}$

$$\text{so } (e^{-t^2/2a} u)' = -\frac{1}{a} f'$$

$$e^{-t^2/2a} u = -\frac{1}{a} \int f' dt = -\frac{f}{a} + c$$

$c=0$  to match IC.

$$u(t) = -\frac{f(t)}{a} e^{t^2/2a}$$

This is #11  
on p. 245