MATH 46 WORKSHEET: Dimensional analysis

W 3/28/07

A) Consider the drag force F on a sphere of radius a, moving at speed v in fluid density ρ.

	F	a	v	ρ
M	1			1
L	1	1	1	−3
T	−2		−1	

i) Fill in the matrix (if don't know dim's of F, think of Newton's (2nd Law), and decide how F depends on the other parameters: $F = c\rho a^2 v^2$

ii) What is π, the dimensionless quantity formed from F, a, v, ρ? $\pi = \frac{F}{\rho a^2 v^2}$

iii) What linear algebra operation did you do get π? nontrivial lin. comb. of columns which vanishes, ie if M is matrix find $\vec{x}$ s.t. $M\vec{x} = \vec{0}$, ($\vec{x}$ is in null space)

B) Consider a pulse of energy e released at the origin at time $t=0$. The medium has heat capacity c (energy per volume per degree), and thermal conductivity κ (power per length per time). The temperature at distance r and time t is u. (Assume $u=0$ everywhere before the pulse)

	e	r	t	u	c	κ
E	1				1	1
L		1			−3	−1
T			1			−1
Θ				1	−1	−1

i) Using fundamental units energy (E), length (L), time (T) and temperature (Θ), Fill in the dimensions of the $m=6$ quantities in the problem.

ii) Find $p=2$ independent dimensionless quantities. Since there's freedom, choose

π_1 to not involve u: $\pi_1 = \dots \frac{r^2 c}{t\kappa}$

π_2 to not involve r: $\pi_2 = \dots \frac{ce^2}{\kappa^3 t^3 u^2}$ or $\frac{(\kappa t)^{3/2} u}{c^{1/2} e}$ (power $-1/2$)

iii) π Theorem tells us $F(\pi_1, \pi_2) = 0$ so $\pi_2 = g(\pi_1)$ (useful for us since u has power 1)

From this get an expression $u = \frac{(ce^{1/2})}{(\kappa t)^{3/2}} g\left(\frac{cr^2}{\kappa t}\right)$

iv) If $r=0$ how does u scale with t? $u = \frac{ce^{1/2}}{(\kappa t)^{3/2}} g(0) = kt^{-3/2}$

v) In general how can you get p from the linear algebra properties of the matrix? $p = m - \text{rank}(M)$