

Final Exam

Math 3

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Name: _____

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Instructions: The Final Exam is multiple choice. You are not allowed to use calculators, books, or notes of any kind. All your answers to the questions must be marked on the Scantron form provided. Take a moment now to **print your name and section clearly on your Scantron form**, and on your exam booklet. You may write on the exam, but you will only receive credit for what you write on the Scantron form. At the end of the exam you must turn in both your Scantron form, and your exam booklet. There are 30 multiple choice problems each worth 5 points, for a total of 150 points. Check to see that you have 13 pages of questions. Good luck. We have enjoyed working with you this term and hope to see you again in the future.

MULTIPLE-CHOICE

1. The domain of the function $f(x) = \ln(\ln x)$ is
 - (a) $(0, \infty)$
 - (b) $(1, \infty)$
 - (c) $(0, 1)$
 - (d) $(-\infty, \infty)$
 - (e) None of the above.

2. The domain of the function $f(x) = \arccos\left(\frac{x}{3} + 1\right)$ is
 - (a) $[-6, 0]$
 - (b) $(-\infty, \infty)$
 - (c) $[-4, 2]$
 - (d) $[-3, 3\pi - 3]$
 - (e) None of the above.

3. Let $f(x) = e^{2x} + 1$ and let $g(x) = e^{2x+2} - 3$. Starting with the graph of f , one obtains the graph of g by:

- (a) Shifting down 3 and right 1.
- (b) Shifting down 3 and left 2.
- (c) Shifting down 4 and left 2.
- (d) Shifting down 4 and left 1.
- (e) None of the above.

4. The inverse of the function $y = \frac{1 - 2x}{1 + x}$ is:

- (a) $y = -2 \ln(1 + x)$
- (b) $y = \frac{-4x - 3}{(1 + x)^2}$
- (c) $y = -\frac{1 + x}{1 - 2x}$
- (d) $y = \frac{1 - x}{2 + x}$
- (e) The function does not have an inverse.

5. Suppose that you have data points $(0, 1)$, $(1, 3)$, and $(2, 9)$. Let

$$f(x) = 3x + 1,$$

$$g(x) = 4x,$$

$$h(x) = 2x^2.$$

Among these three functions, the one which minimizes the least square error is:

- (a) f alone.
- (b) g alone.
- (c) h alone.
- (d) Both g and h .
- (e) All three give the same least square error.

6. The limit $\lim_{x \rightarrow \infty} \frac{6x^2 + 5x}{2x^3 + 1}$ equals:

- (a) 0
- (b) $2/3$
- (c) 3
- (d) 6
- (e) Does not exist.

7. Let $f(x) = \frac{\sqrt{4x^2 + 1}}{x - 5}$. The horizontal and vertical asymptotes of f are:

- (a) $x = 5, y = 2, y = -2$
- (b) $x = 5, y = 2$
- (c) $x = 5, y = 4$
- (d) $x = 1/2, y = 5$
- (e) No choice above gives the complete and correct list of horizontal and vertical asymptotes.

8. The limit $\lim_{h \rightarrow 0} \frac{\ln(e + h) - 1}{h}$ equals:

- (a) 1
- (b) 0
- (c) e
- (d) $1/e$
- (e) Does not exist.

9. At $x = 2$, the function $f(x) = \frac{x^2 - 4}{x - 2}$

- (a) is continuous.
- (b) has a local maximum.
- (c) is differentiable.
- (d) has an inflection point.
- (e) has a removable discontinuity.

10. For what value of k is the following function differentiable at $x = 2$?

$$f(x) = \begin{cases} x^2 + kx + 2 & \text{if } x < 2 \\ \ln(x - 1) & \text{if } x \geq 2 \end{cases}$$

- (a) -3
- (b) -2
- (c) 0
- (d) $e + 1$
- (e) No value of k .

11. The slope of the tangent line to the graph of $f(x) = \arctan(2^x)$ at $x = 1$ is:

(a) $\frac{2 \ln 2}{5}$

(b) $\frac{2}{5}$

(c) $\frac{2}{\sqrt{5}}$

(d) Not defined, since $x = 1$ is not in the domain of f .

(e) None of the above.

12. The tangent line to the curve $y = e^{x^2-1} + 8\sqrt[4]{x}$ at the point with $x = 1$ is given by:

(a) $y = 9x - 1$

(b) $y = 3x + 6$

(c) $y = 4x - 4$

(d) $y = 4x + 5$

(e) None of the above.

13. The slope of the line tangent to the curve $xy^3 + x^2y + 2x = 10$ at the point $(2, 1)$ is:

- (a) $-3/4$
- (b) 10
- (c) $1/2$
- (d) $-7/10$
- (e) None of the above.

14. Which of the following statements is true.

- (a) If f is continuous at a , then f is differentiable at a .
- (b) If f is continuous on $[a, b]$, then f is Riemann integrable on $[a, b]$.
- (c) If f is continuous on $[a, b]$, then there exists a point x in $[a, b]$ such that $f(x) = 0$.
- (d) If f is continuous on $[a, b]$ and differentiable on (a, b) , then there exists a point x in (a, b) such that $f'(x) = 0$.
- (e) If f is differentiable on $[a, b]$, then $\int_a^b f'(x) dx = f(b)$.

15. The linearization of the function $f(x) = x^3 - x + 4$ at the point $a = 1$ intersects the x -axis at:

- (a) -3
- (b) -1
- (c) 0
- (d) 1
- (e) None of the above.

16. On which intervals is the function $f(x) = \frac{x^2 - 8}{x - 3}$ decreasing?

- (a) $(-\infty, 3) \cup (3, \infty)$
- (b) $(-4, 3)$
- (c) $(3, 4)$
- (d) $(-4, 3) \cup (3, 4)$
- (e) $(2, 3) \cup (3, 4)$

17. How many inflection points does $f(x) = \frac{x^5}{5} + \frac{x^3}{3} + 7$ have?

- (a) 0
- (b) 1
- (c) 2
- (d) 3
- (e) More than 3.

18. Assume that a particle is moving along the x -axis and the acceleration at time t is given by $a(t) = 6t$. Suppose the position at time $t = 1$ is 2 and the velocity at time $t = 1$ is 1. The position of the particle at time $t = 2$ is:

- (a) 6
- (b) 7
- (c) 12
- (d) 13
- (e) None of the above.

19. The solution to the initial value problem $y' = x^2y - 2x^2$, $y(0) = 1$ is

- (a) $y = 2 - e^{x^3/3}$
- (b) $y = e^{x^3/3}$
- (c) $y = x^3 + x^2 + 1$
- (d) $y = \ln(x^2 + 1) + 1$
- (e) None of the above.

20. Consider the initial value problem $y' = 2xy + 1$, $y(0) = 2$. Using Euler's method to find an approximation of the solution curve that starts at $(0, 2)$ using steps of size $1/2$, what is the approximate value of $y(1)$?

- (a) 1
- (b) 2
- (c) $17/4$
- (d) $5/2$
- (e) None of the above.

21. Suppose that the number of rabbits in a certain farm follows an exponential model (that is, the growth rate of the population is proportional to its size). Assuming that there were 150 rabbits one year ago, and that there are 300 today, how many rabbits are there going to be a year from today?

- (a) 400
- (b) 450
- (c) 600
- (d) 750
- (e) 900

22. Suppose that you want to approximate the integral $\int_a^b f(x) dx$ and you break up $[a, b]$ as $a = x_0 < x_1 < \dots < x_n = b$ where each interval $[x_{i-1}, x_i]$ has equal length h . If you use rectangles whose height is the given by the value of the function at the right-endpoint of each interval, you would calculate:

- (a) $\frac{h}{2}(f(x_0) + f(x_n)) + \sum_{i=1}^{n-1} 2 \cdot f(x_i)$
- (b) $\sum_{i=1}^n f(x_{i-1}) \cdot h$
- (c) $\sum_{i=1}^n f(x_i) \cdot h$
- (d) $\sum_{i=1}^n f'(x_i)$
- (e) $f(b) - f(a)$

23. The integral $\int_{-\pi/2}^{\pi/2} (x - x^2 \sin x) dx$ is:

- (a) 0
- (b) $\pi/2$
- (c) $\pi^2/4$
- (d) $\pi/4$
- (e) None of the above.

24. The value of $\int_1^e \frac{\ln x}{x} dx$ is

- (a) $1/2$
- (b) 1
- (c) e
- (d) -1
- (e) None of the above.

25. The integral $\int \frac{1}{\sqrt{5-x^2}} dx$ is:

(a) $\arcsin \frac{x}{\sqrt{5}} + C$

(b) $\arcsin \frac{x}{5} + C$

(c) $\frac{1}{\sqrt{5}} \arcsin \frac{x}{\sqrt{5}} + C$

(d) $\arctan \frac{x}{5} + C$

(e) $2\sqrt{5-x^2} + C$

26. The area of the region below the curve $y = \sqrt{4-x^2}$ and above the interval $[-2, 0]$ is:

(a) $\arcsin(-2)$

(b) $\pi/2$

(c) π

(d) 2π

(e) 0

27. $\frac{d}{dx} \int_{x^3}^1 e^{\sin t} dt$ equals

(a) $-e^{\sin x} \cos x$

(b) $-3x^2 e^{\sin x^3}$

(c) $e^{\sin x}$

(d) $-e^{\sin x^3}$

(e) None of the above.

28. The area of the region bounded by the curves $y = x^3 - 3x^2 + 1$ and $y = x^2 - 4x + 1$ is

(a) $-3/4$

(b) $3/4$

(c) $-4/3$

(d) $4/3$

(e) None of the above.

29. The length of the arc of the curve $y = \frac{x^2}{2} - \frac{\ln x}{4}$ from $(1, \frac{1}{2})$ to $(e, \frac{e^2-1}{2})$ is:

- (a) $\frac{e^2}{2} - \frac{1}{4}$
- (b) $e + 1$
- (c) $\frac{1}{2e}$
- (d) $e + \frac{1}{4}$
- (e) None of the above.

30. Consider the following function, which gives the rainfall (in inches per hour) over an interval of 3 hours.

$$f(x) = \begin{cases} 0, & x \leq 0 \\ 2, & 0 < x \leq 1 \\ 3, & 1 < x \leq 3 \\ 0, & x > 3. \end{cases}$$

What is the x -coordinate of the centroid of the precipitation?

- (a) 13 hours.
- (b) $13/8$ hours.
- (c) 8 hours.
- (d) $8/13$ hours.
- (e) None of the above.