Math 3: Fall 2007
Exam 1 solutions

1. The number $\sin\pi + \ln 1 + e^0$ can be simplified to

(a) 0

(b) $\sqrt{2}$

(c) -1

(d) 2

(e) none of the above

Answer: (e) (the number can be simplified to 1)

2. What symmetry does the graph of $y = x^2 - 6x + 10$ have?

(a) It is an even function and so is symmetric about the y-axis.

(b) It is an odd function and so is symmetric about the origin.

(c) It is symmetric about the line $y = 3$.

(d) It is symmetric about the line $x = 3$.

(e) It has no symmetry.

Answer: (d)

3. Consider the function $f(x) = 3x^2 - 4x + 1$. The equation of the tangent line to the graph of $f(x)$ at $(0, 1)$ is

(a) $y = 6x - 4$

(b) $y = 6x + 1$

(c) $y = 2x + 1$

(d) $y = -4x + 1$

(e) none of the above

Answer: (d)

4. The limit

$$\lim_{x \to 7} \frac{x - 7}{|x - 7|}$$

is

(a) 1

(b) -1

(c) 0

(d) does not exist, but the function tends to ∞

(e) does not exist

Answer: (e)

5. Which of the following describes the behavior of the function

$$f(x) = \begin{cases} 3(x-2)^2 - 5 & x \neq 2, \\ 0 & x = 2 \end{cases}$$

at $x = 2$?

(a) $\lim_{x \to 2} f(x)$ does not exist.

(b) $\lim_{x \to 2} f(x)$ exists but f is not continuous at $x = 2$.

(c) f is continuous but not differentiable at $x = 2$.

(d) f is differentiable but not continuous at $x = 2$.

(e) None of the above.

Answer: (b)

6. Suppose $f(x)$ is a continuous function with domain the closed interval $[-1, 3]$. Suppose too that $f(-1) = 2$ and $f(3) = -2$.

(a) There must be some number c with $-1 < c < 3$ with $f(c) = 3$.

(b) There must be some number c with $-1 < c < 3$ where f is not differentiable.

(c) There must be some number c with $-1 < c < 3$ with $f(c) = e$.

(d) There must be some number c with $-1 < c < 3$ with $f(c) = 1/\pi$.

(e) The given information is not enough to conclude any of the above.

Answer: (d)

7. The slope of the tangent line to $f(x) = (x^3 - x + 1)^{11}(2x^2 + x - 3)^7$ at $x = 1$ is

(a) $(33x^2 - 11)(x^3 - x + 1)^{10}(28x + 7)(2x^2 + x - 3)^6$

(b) 0

(c) does not exist

(d) 22

(e) none of the above

Answer: (b)

8. The limit

$$\lim_{x\to\infty} \frac{3x^3 - 4x^2 + 5}{7x^5 + 9x^4 - 3x}$$

is

(a) 0

(b) $-\frac{5}{3}$

(c) $\frac{3}{7}$

(d) does not exist, but the function tends to ∞

(e) does not exist

Answer: (a)

9. The inverse of the function $y = \frac{1+e^x}{1-e^x}$ is

(a) $y = \frac{x-1}{x+1}$

(b) $y = \frac{x-1}{e(x+1)}$

(c) $y = \ln\left(\frac{x+1}{x-1}\right)$

(d) $y = \ln\left(\frac{x-1}{x+1}\right)$

(e) The function has no inverse

Answer: (d)

10. The derivative of

$$f(x) = x \sin\sqrt{x}$$

is

(a) $\sin\sqrt{x} + x\cos\sqrt{x}$

(b) $\frac{1}{2}x\cos\left(\frac{1}{\sqrt{x}}\right)$

(c) $\sin\sqrt{x} + \frac{1}{2}\sqrt{x}\cos\sqrt{x}$

(d) $\sin\sqrt{x} + x\cos\sqrt{x} + \frac{1}{2}x\sin\left(\frac{1}{\sqrt{x}}\right)$

(e) none of the above

Answer: (c)

NON-MULTIPLE CHOICE. PLEASE SHOW ALL YOUR WORK.

You do not need to use the limit definition of the derivative for any of these problems. You may use the differentiation rules.

11. A falling stone travels $4.9t^2$ meters in t seconds (ignoring air resistance) and it continues to fall for 12 seconds.

(a) What is the stone's average speed over the first 10 seconds?

The function $f(t) = 4.9t^2$ gives the distance travelled by the falling stone. The average speed of the stone over the time interval $[0, 10]$ is

$$\frac{f(10) - f(0)}{10 - 0} = \frac{4.9(10)^2 - 4.9(0)^2}{10} = 49 \text{ meters per second.}$$

(b) What is the stone's instantaneous speed at the 10-second mark?

The instantaneous speed of the stone is given by the derivative of f. Since $f'(t) = 9.8t$, the instantaneous speed at the 10-second mark is $f'(10) = 98$ meters per second.

(c) What is the stone's instantaneous acceleration at the 7-second mark?

The instantaneous acceleration of the stone is given by the second derivative of f. Since $f''(t) = 9.8$, the instantaneous acceleration at the 7-second mark is 9.8 meters per second squared.

12. Consider the following table of experimental data points.

x	y
2	-2
3	2
4	2
6	10

(a) For each of the two lines

$$L_1(x) = 2x - 6 \quad \text{and} \quad L_2(x) = 3x - 8,$$

compute the sum of the squared errors in comparison with the experimental data.

The sum of squared errors for line L_1 is

$$(-2-(2\cdot 2-6))^2+(2-(2\cdot 3-6))^2+(2-(2\cdot 4-6))^2+(10-(2\cdot 6-6))^2$$
$$= 0^2 + 2^2 + 0^2 + 4^2 = 20.$$

The sum of squared errors for line L_2 is

$$(-2-(3\cdot 2-8))^2+(2-(3\cdot 3-8))^2+(2-(3\cdot 4-8))^2+(10-(3\cdot 6-8))^2$$
$$= 0^2 + 1^2 + (-2)^2 + 0^2 = 5.$$

(b) Which line gives the better fit to the data?

Line L_2 gives the better fit, since its sum of squared errors is less than the sum of squared errors for L_1.

13. Consider the function

$$f(x) = \frac{x^2 + 5x + 4}{x^2 - 16}.$$

(a) What is the domain of f?

We can factor the function as

$$f(x) = \frac{(x+1)(x+4)}{(x+4)(x-4)},$$

so f is defined for all x except $x = 4$ and $x = -4$. Then the domain of f is $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$.

(b) For each point c not in the domain of f, does f have a continuous extension to $x = c$? If so, what value should be assigned to f at c?

The function f has a continuous extension to $x = -4$, because

$$\lim_{x \to -4} \frac{(x+1)(x+4)}{(x+4)(x-4)} = \lim_{x \to -4} \frac{(x+1)}{(x-4)} = \frac{3}{8}.$$

If we define $f(-4) = \frac{3}{8}$, then f will be continuous at $x = -4$.

But f does not have a continuous extension to $x = 4$, because $\lim\limits_{x \to 4} f(x)$ does not exist. Note that

$$\lim_{x \to 4^+} \frac{(x+1)(x+4)}{(x+4)(x-4)} = \lim_{x \to 4^+} \frac{(x+1)}{(x-4)} = \infty$$

and

$$\lim_{x \to 4^-} \frac{(x+1)(x+4)}{(x+4)(x-4)} = \lim_{x \to 4^-} \frac{(x+1)}{(x-4)} = -\infty.$$

Since the limit does not exist, there is no way to define $f(4)$ so that f is continuous at $x = 4$.

(c) Describe all vertical asymptotes that occur in the graph of f.

From the left and right sided limits in part (b), we see that f has a vertical asymptote at $x = 4$.